Elementary Mathematics Endorsement Specs Competencies & Requirements

Purpose

The purpose of the Elementary Mathematics Endorsement program is to ensure that practicing teachers gain the mathematical content knowledge and pedagogical content knowledge (PCK) (Ball, Thames, & Phelps, 2008) needed to teach mathematical concepts to students in Grades K-6. The Elementary Mathematics Endorsement may be added to a Utah Educator License with an Early Childhood, Elementary, or Special Education area of concentration. The Elementary Mathematics Endorsement deepens knowledge of K-6 content and provides an understanding of how the K-6 content relates to the middle school mathematics content.

Please note: The Elementary Mathematics Endorsement does NOT endorse educators to teach in a middle school setting. A Middle School Mathematics Endorsement is required to teach 7th and 8th grade mathematics.

Prerequisites

To be eligible for this endorsement, candidates must meet the following prerequisites:

- Have a Utah Educator License that contains an Elementary, Early Childhood, or Special Education area of concentration.

ENDORSEMENT REQUIREMENTS:

The Elementary Mathematics Endorsement has the following 6 content areas.

*Please Note: If an applicant chooses the university coursework pathway, the courses for Geometry and Measurement and Data Analysis and Problem Solving must be taken **separately.** If an applicant chooses the competency pathway, Geometry and Measurement is **combined** with Data Analysis and Problem Solving.*

1. Numbers and Operation
2. Rational Numbers and Proportional Reasoning
3. Geometry and Measurement
4. Data Analysis and Problem Solving
5. Algebraic Reasoning
6. Assessment and Intervention

A minimum of 18 credit hours is required to meet the Elementary Mathematics Endorsement guidelines of the Utah State Board of Education. All six required courses must be completed. Coursework older than eight years will not be accepted. Completed courses must have grades of C or better.

For the Competency Pathway

To establish competencies in any of the 5 requirement areas:
- Use the Demonstration of Competencies for Elementary Math Endorsement form.
  - This form must be completed and signed by an Elementary Mathematics Representative
    - For example: Direct Supervisor, Elementary Mathematics District or Charter Specialist, or an Elementary Mathematics Educator Leader or Coach.

For the Microcredential Pathway:

- To learn more about microcredentials in general:
  - Utah Education Network Microcredentials
  - MIDAS Education

- To learn more about microcredentials for Elementary Mathematics:
  - MIDAS Education Microcredentials

Overview of Requirement Areas and Approved Competency Paths for the Elementary Mathematics Endorsement

Requirement Area #1 - Numbers and Operations

University Courses Pathway

- 3 credit college/university course
  - Mathematics for Teaching K-6: Numbers and Operations

Demonstration of Competencies Pathway

Teacher Demonstrated Competencies for Requirement Area #1:

- Understand and respond to developmentally appropriate learning trajectories related to each of the listed concepts by planning and implementing instruction based on the Standards for Mathematical Practice and Effective Mathematics Teaching Practices.

- Select, use, and adapt mathematics curricula and teaching materials including the integration of mathematical tools and technology.

- Use and analyze formative and summative assessments to determine where students are in learning trajectories related to early number concepts.

For the Following Student Learning Concepts:

Early Number Concepts include:
- Make non-quantified comparisons.
- Understand containment.
- Explore notions of quantity and measurement
- Understand one-to-one correspondence, cardinality, meaningful counting, and ordinality.
- Compose and decompose whole numbers.

Place Value includes:
- A comprehensive repertoire of interpretations
  - visual, physical, symbolic, contextual, verbal

Whole Number Addition and Subtraction includes:

- A comprehensive repertoire of interpretations and representations
    - visual, physical, symbolic, contextual, verbal
- Properties of addition and common ways they can be applied.
- A variety of methods to solve basic and multi-digit addition and subtraction problems.
- Understanding the relationship between addition and subtraction.

Whole Number Multiplication and Division includes:

- A comprehensive repertoire of interpretations
- Properties of multiplication and the distributive property and common ways they can be applied.
- A variety of methods to solve basic and multi-digit multiplication and division problems.
- Understanding the relationships between the four basic operations and how they relate to each other.

Computation

- Extend understanding of the four operations to perform multi-digit calculations.
- Use: mental math, non-standard ways commonly created by students, standard algorithms, informal reasoning
- Connect ideas, strategies, and representations to build procedural fluency from conceptual understanding.
- Combine number sense and operation sense to invent algorithms.

Multiplicative Arithmetic

- Work with and understand:
    - odd and even numbers
    - factors and factorization
    - multiples
    - primes, prime factorization (Fundamental Theorem of Arithmetic)
    - composite numbers
    - least common multiple and greatest common factor

Fluency includes:

- Use strategies involving composition and decomposition to be flexible, efficient, and accurate with solving problems (Parrish, 2014; Russell, 2000).
- Understand "Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from

the use of strategies … fluency will be a mixture of these kinds of thinking which may differ across students."

Microcredential Pathway

- Microcredential Stack ID 2934
    - Elementary Mathematics Endorsement: Numbers and Operations – Procedural Fluency
- Microcredential Stack ID 2935
    - Elementary Mathematics Endorsement: Numbers and Operations – Number Concepts
- Microcredential Stack ID 2942
    - Elementary Mathematics Endorsement: Numbers and Operations – Place Value Concepts
- Microcredential Stack ID 2944
    - Elementary Mathematics Endorsement: Numbers and Operations – Whole Number Addition and Subtraction
- Microcredential Stack ID 2946
    - Elementary Mathematics Endorsement: Numbers and Operations – Whole Number Multiplication and Division

Requirement Area #2 - Rational Numbers and Proportional Reasoning

University Course Pathway

- 3 credit college/university course
    - Rational Numbers and Proportional Reasoning

Demonstration of Competencies Pathway

Teacher Demonstrated Competencies for Requirement Area #2:

- Understand and respond to developmentally appropriate learning trajectories related to each of the listed concepts by planning and implementing instruction based on the Standards for Mathematical Practice and Effective Mathematics Teaching Practices.

- Select, use, and adapt mathematics curricula and teaching materials including the integration of mathematical tools and technology.

- Use and analyze formative and summative assessments to determine where students are in learning trajectories related to early number concepts.

For the Following Student Learning Concepts:

Early Fraction Concepts include:
- Create fair shares and equal parts.
- Understand fractional language.
- Use shape and number to decompose and compose a whole into equal shares, partition shapes into fractional pieces.
- Compare sizes of units
- Use visual fraction area models.

Fraction Concepts include:
- Express equal shares using fractional notation.
- Understand fractions as numbers.
- Use visual fraction length models (e.g., fraction strips, number lines, bar/strip models, rulers) and visual fraction set models.
- Use unit fractions to compose and decompose shape and number.

Decimal and Percent Concepts include:
- Connect fractions to decimals using place value.
- Understand that decimals represent quantities between whole numbers and there are infinite numbers between any two numbers (theorem name).
- Use visual decimal models.
- Extend understanding of the place value system including the role of the decimal point.
- Express fractional amounts using equivalent fractions, decimal notation, and percent notation.
- Understand percent as hundredths or a standardized ratio with a denominator of 100.

Comparison and Equivalence with Rational Numbers includes:
- Know relative sizes of fractions using benchmark fractions.
- Recognize that comparisons are valid only when the two fractions refer to the same whole; attend to units.

- Compare fractions by reasoning about the size of the fractions, using visual models, and by finding common numerators or denominators.
- Recognize and generate equivalent fractions.
- Convert flexibly between numerical and visual representations of fractions, decimals, and percents.
- Compare and order rational numbers.
- Use place-value and scientific notation to order numbers, estimate, and represent order of magnitude.

Addition and Subtraction with Rational Numbers includes:
- Apply and extend whole number strategies to rational numbers including:
  - A comprehensive repertoire of representations
    - visual, physical, symbolic, contextual, verbal
  - Properties of addition and common ways they can be applied.
  - A variety of methods to solve addition and subtraction problems.
  - Understanding the relationship between addition and subtraction to add and subtract rational numbers.
- Develop context for adding and subtracting rational numbers.
- Add and subtract fractions and decimals using reasoning, visual models, number lines, and mental strategies.
- Understand that same-sized pieces must be used to combine and separate pieces. and this may include finding common denominators and aligning decimals points
- Develop and analyze algorithms for adding and subtracting with rational numbers.

Multiplication and Division with Rational Numbers includes:
- Apply and extend whole number strategies to rational numbers including:
  - A comprehensive repertoire of representations
    - visual, physical, symbolic, contextual, verbal
  - Properties of multiplication and the distributive property including common ways they can be applied.
  - A variety of methods to solve multiplication and division problems.
  - Understanding the relationship between multiplication and division to multiply and divide rational numbers.
  - Understanding the relationships between the four basic operations and how they relate to each other.
- Develop context for multiplying and dividing rational numbers.
- Interpret a fraction as division of the numerator by the denominator.

- Multiply and divide fractions and decimals using reasoning, visual models, number lines, and mental strategies.
- Understand when multiplying a given number by a fraction greater than one results in a product greater than the given number and when multiplying a given number by a fraction less than one results in a product smaller than the given number.
- Develop and analyze algorithms for multiplying and dividing rational numbers.

Proportional Relationships includes:
- Understand ratio concepts, ratio language, and types of ratios.
- Understand rate as a special type of ratio that involves two different units and how they relate to each other.
- Understand unit rate and its relationship to proportionality.
- Attend to units in all rate and ratio situations.
- Compose (iterate) and decompose (partition) ratios using multiple representations.
- Use tables, graphs, and equations to generate equivalent ratios in context.
- Explore and understand purpose of scale and levels of accuracy in measurement needed in context.

Microcredential Pathway
- Microcredential Stack ID 2948
  - Elementary Mathematics Endorsement: Rational Numbers and Proportional Reasoning – Multiplication and Division with Rational Numbers
- Microcredential Stack ID 2950
  - Elementary Mathematics Endorsement: Rational Numbers and Proportional Reasoning – Proportional Relationships
- Microcredential Stack ID 2951
  - Elementary Mathematics Endorsement: Rational Numbers and Proportional Reasoning – Comparison and Equivalence with Rational Numbers
- Microcredential Stack ID 2953
  - Elementary Mathematics Endorsement: Rational Numbers and Proportional Reasoning – Decimal and Percent Concepts
- Microcredential Stack ID 2962
  - Elementary Mathematics Endorsement: Rational Numbers and Proportional Reasoning - Fractions
- Microcredential Stack ID 2963

- o Elementary Mathematics Endorsement: Rational Numbers and Proportional Reasoning – Early Fraction Concepts
- Microcredential Stack ID 2964
  - o Elementary Mathematics Endorsement: Rational Numbers and Proportional Reasoning – Addition and Subtraction with Rational Numbers

Requirement Area #3 - Geometry, Measurement, and Data

University Course Pathway

- 3 credit college/university course for each course.
  - o Geometry and Measurements
  - o Data and Problem Solving
    - ▪ Classes must be taken separately.

Demonstration of Competencies Pathway

Teacher Demonstrated Competencies for Requirement Area #3:

- Understand and respond to developmentally appropriate learning trajectories related to each of the listed concepts by planning and implementing instruction based on the Standards for Mathematical Practice and Effective Mathematics Teaching Practices.

- Select, use, and adapt mathematics curricula and teaching materials including the integration of mathematical tools and technology.

- Use and analyze formative and summative assessments to determine where students are in learning trajectories related to early number concepts.

For the Following Student Learning Concepts:

Visualization, Composition, and Decomposition of Geometric Shapes includes:

- Visualization
  - o Recognize shapes in the environment, develop relationships between two- and three-dimensional objects, and draw and recognize objects from different viewpoints and perspectives (Van de Walle, Karp, & Bay-Williams, p. 403)

- Composition and Decomposition
  - Assemble a geometric figure by joining together various component figures.
  - Decompose a geometric figure into pieces.
  - Rearrange decomposed components of figures to create other shapes.

Understand and Implement Concepts of Measurement includes:

- Understand Measurement Concepts
  - Know that measurement involves:
    - An attribute to be measured.
    - A measurement system.
    - Units
    - Tools
  - Compare measurements directly and indirectly.
  - Estimate, use benchmark measurements, and measure with precision.
  - Explore and understand purpose of scale and levels of accuracy in measurement needed in context.
- Geometric Measurement
  - Quantify size and shape by attaching a numerical quantity to 1-D, 2-D, and 3-D geometric figures.
  - Describe relationships of the measurements taken in different units and convert measurements.
- 1-D Measurements
  - Length
  - Perimeter
- 2-D Measurement
  - Area / Surface Area
  - Angles
- 3-D Measurement
  - Volume

Analyze Characteristics and Properties of 2-D and 3-D Figure includes:

- Analyze Characteristics and Properties of 2-D and 3-D Figures
  - Classify and sort geometric figures by characteristics and properties.
  - There are non-defining (color) and defining attributes.
- Construct and Support Viable Arguments

- o Model, draw, and create shapes, angles, or lines attending to properties of shapes; develop a sequence of justified statements that show truth.

Coordinate Plane includes:

- Compose and understand the coordinate plane as vertical and horizontal number lines.
- Use positional language to identify location.
- Graph points (coordinates) on a coordinate plane.
- Connect algebraic expressions to geometric objects and relationships.

Data Analysis and Problem Solving includes:

- o The Nature and Uses of Data
  - Pose questions.
    - Understand how to formulate appropriate questions for types of data.
    - Understand the relationship between posed questions and methods for data analysis and display.
    - Discern between statistical questions and non-statistical questions.
    - Understand common student fallacies for question development.
  - Visualize, organize, and display data.
    - Create visual and graphical representations to solve problems.
    - Differentiate between tools for collecting (observations, surveys, and experiments) and organizing data.
    - Use technology to generate, collect, understand, and organize data.
    - Differentiate how to display different types of single-variable data using appropriate displays.
    - Communicate essential components of displays using labels, units, scales, keys, etc.
  - Analyze, interpret, summarize data.
    - Answer questions about data
    - Solve problems with data.
    - Analyze data using measures of center, spread/variation and shape; relate center and spread to shape.

▪ Understand how different representations communicate different aspects of data and influence interpretation.
▪ Use inferences to make decisions based on data.

Microcredential Pathway

- Microcredential Stack ID 2923
  - o Elementary Mathematics Endorsement: Geometry Measurement, and Data – Measurement
- Microcredential Stack ID 2924
  - o Elementary Mathematics Endorsement: Geometry Measurement, and Data – Visualization, Composition, and Decomposition of Geometric Shapes
- Microcredential Stack ID 2925
  - o Elementary Mathematics Endorsement: Geometry Measurement, and Data – Analyzing Characteristics and Properties of 2-D and 3-D Figures
- Microcredential Stack ID 2926
  - o Elementary Mathematics Endorsement: Geometry Measurement, and Data – Coordinate Planes
- Microcredential Stack ID 2933
  - o Elementary Mathematics Endorsement: Geometry Measurement, and Data – Data

Requirement Area #4 - Algebraic Reasoning

University Course Pathway

- 3 credit college/university course (Algebraic Reasoning)

Demonstration of Competencies Pathway

Teacher Demonstrated Competencies for Requirement Area #4:

- Understand and respond to developmentally appropriate learning trajectories related to each of the listed concepts by planning and implementing instruction based on the Standards for Mathematical Practice and Effective Mathematics Teaching Practices.

- Select, use, and adapt mathematics curricula and teaching materials including the integration of mathematical tools and technology.

- Use and analyze formative and summative assessments to determine where students are in learning trajectories related to early number concepts.

For the Following Student Learning Concepts:

Algebraic Notation, Equality, Inequality, and Relational Thinking includes:
- Understand the meaning of the equal sign as relational.
- Recognize the difference between expressions, equations, and inequalities.
- Understand equivalent expressions as equations.
- Represent unknown values with symbols and variables.
- Model mathematical and real-world problems using algebraic equations and inequalities.
- Use algebraic notation appropriately including the equal sign, inequality symbols, symbols for operations, letters as variables, grouping symbols.
- Evaluate expressions and solve equations and inequalities.
- Understand the need for and use of order-of-operations conventions.
- Apply and extend previous understandings of arithmetic to algebraic expressions involving variables and whole number exponents.
- Use the process of substitution of numbers into variable expressions and find the solution set of an algebraic equation or inequality.

Algebraic Properties and Conjectures includes:
- Understand how properties of operations may be used in computations and to deduce the correctness of algorithms.
- Understand conjectures arise when one notices a pattern that holds true for many cases.
- Make conjectures about classes of numbers.
- Make conjectures about rules for carrying out specific computational procedures.
- Make conjectures about inverse operations and other relationships among addition, subtraction, multiplication, and division.
- Connect conjectures to existing rules, properties, and definition.

Microcredential Pathway

- Microcredential Stack ID 2911
  - Elementary Mathematics Endorsement: Algebraic Reasoning – Algebraic Notation, Equality, Inequality, and Relational Thinking

- Microcredential Stack ID 2913
  - Elementary Mathematics Endorsement: Algebraic Reasoning - Patterns
- Microcredential Stack ID 2919
  - Elementary Mathematics Endorsement: Algebraic Reasoning – Algebraic Properties and Conjectures

Requirement Area #5 - Assessment and Intervention

University Course Pathway

- 3 credit college/university course (Assessment and Intervention)

Demonstration of Competencies Pathway

Teacher Demonstrated Competencies for Requirement Area #5:

Multi-tiered System of Supports including:

- Gain deeper assessment literacy including being able to create, select, and effectively use classroom assessments and being able to select and effectively interpret and use results from assessments (Kobett, B. M., & Wray, J. A., 2016).
- Understand and implement the critical components of the UT MTSS for Math
- Understand and implement effective intervention strategies.

Equitable Assessment Practices

- Understand when access and equity have been successfully addressed, student outcomes—including achievement on a range of mathematics assessments, disposition toward mathematics, and persistence in the mathematics pipeline—transcend, and cannot be predicted by students' racial, ethnic, linguistic, gender, and socioeconomic backgrounds (NCTM, 2014. Access and equity in mathematics education).

Microcredential Pathway

- Microcredential Stack ID: 2921
  - Elementary Mathematics Endorsement: Assessment and Intervention – Equitable Assessment Practices
- Microcredential Stack ID: 2922
  - Elementary Mathematics Endorsement: Assessment and Intervention – Multi-Tiered System of Supports (MTSS)