MATHEMATICS

UTAH CORE **GUIDES** GRADE **5**

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USBE ADA Compliant March 2021

Write and interpret numerical expressions (Standards 5.OA.1–2)

Standard 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Concepts and Skills to Master

- Understand and explain the steps in the order of operations
- Understand and explain the purposes of parentheses, brackets, or braces in numerical expressions
- Understand and explain the difference between numerical expressions and numerical equations
- Solve multi-step problems using parentheses, brackets, or braces
- Use a variety of examples to model the importance of grouping symbols. For example: [32 ÷ 4] + [27 ÷ 3] = n. Note: If a student didn't use grouping symbols and didn't understand order of operations, he/she might try to solve the problem going from left to right. Example: 32 ÷ 4 + 27 ÷ 3 = *n* 8 + 27 ÷ 3 = n 35 ÷ 3 = 11 R. 2 (Incorrect Answer) [8] + [9] = 17
- Use physical models, pictures, drawing, diagrams, etc. to represent grouping items using parentheses, brackets, or braces.

Teacher Note: There is no particular significance for when to use parentheses, brackets or braces. The different grouping symbols are an efficient way to keep track of the different parts of a problem. Round parentheses are the most commonly used, but square brackets and curly braces may also be used. This work should be viewed as exploratory rather than for attaining mastery. The numbers in expressions do not need to be limited to whole numbers. This standard builds on third grade knowledge of the order of operations by adding the parentheses, brackets, and braces.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.OA.2 Write and interpret simple numerical expressions **5.OA.3** Generate numerical patterns given two rules, form ordered pairs, and graph on a coordinate plane	**6.EE.1** Write and evaluate numerical expressions involving whole-number exponents **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers **6.EE.3** Apply the properties of operations to generate equivalent expressions **6.EE.4** Identify when two expressions are equivalent **6.NS.4** Use the distributive property to express the sum of two whole numbers with a common factor: 36 + 8 is the same as 4(9 + 2)

Critical Background Knowledge from Previous Grade Levels

- Understand and solve the steps of the order of operations without exponents or parentheses (3.OA.8)
- Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations. Represent these problems using equations with a letter standing for the unknown quantity (4.OA.3)

Academic Vocabulary

expression, parentheses, bracket, brace, operation, order of operations, evaluate

Suggested Models

Evaluate the following numerical expressions.

a. $2 \times 5 + 3 \times 2 + 4$
b. $2 \times (5 + 3 \times 2 + 4)$
c. $2 \times 5 + 3 \times (2 + 4)$
d. $2 \times (5 + 3) \times 2 + 4$
e. $(2 \times 5) + (3 \times 2) + 4$
f. $2 \times (5 + 3) \times (2 + 4)$

Can the parentheses in any of these expressions be removed without changing the value the expression?

Expand the expression below:

17
10 + 7
(2 x 5) + 7
[2 x (30 ÷ 6)] + 7
[2 x (15 x 2 ÷ 6)] + 7

Suggested Strategies

- Solve expressions with and without parentheses to show different answers
- Generate specific answers given a set of four numbers; For example, using 1, 2, 3, 4 find two ways to make 9, two ways to make 7, can you make 26?
- Play target number games in which students write equations using the order of operations to make a target number and explain their reasoning

Image Sources: http://achievethecore.org/coherence-map/#5/24/244/244; https://www.georgiastandards.org/Georgia-Standards/Frameworks/5th-Math-Unit-1.pdf

Write and interpret numerical expressions (Standards 5.OA.1–2)

Standard 5.OA.2 Write and interpret simple numerical expressions.

a. Write simple expressions that record calculations with numbers. *For example, use 2 × (8 + 7) to express the calculation "add 8 and 7, then multiply by 2."*

b. Interpret numerical expressions without evaluating them. *For example, use conceptual understanding of multiplication to interpret 3 × (18939 + 921) as being three times as large as 18932 + 921 without calculating the indicated sum or product.*

Concepts and Skills to Master

- Understand that the word "then" implies one operation happens after another and parentheses are used to indicate the order of operations. Example: "Add 8 and 7, then multiply by 2" can be written as (8 + 7) × 2
- Understand how to write a real-world problem as an expression
- Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921 without having to solve
- Recognize that 3(18,932 + 921) means the same thing as 3 × (18, 932 + 921)
- Write expressions using the correct numerical and symbolic notation in the proper order
- Use numerical and symbolic notation to represent an expression from a problem

Teacher Note: Expressions are a series of numbers and symbols without an equal sign. 4(5 + 3) is an expression. Equations result when two expressions are set equal to one another. 4(5 + 3) = 32 is an equation. Numerical expressions may include whole numbers, decimals, and/or fractions. In fifth grade students are not expected to interpret expressions involving variables. Interpreting variables is reserved for sixth grade in standard 6.EE.2.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols **5.OA.3** Generate numerical patterns given two rules, form ordered pairs, and graph on a coordinate plane	**6.EE.1** Write and evaluate numerical expressions involving whole-number exponents **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers **6.EE.3** Apply the properties of operations to generate equivalent expressions **6.EE.4** Identify when two expressions are equivalent **6.NS.4** Use the distributive property to express the sum of two whole numbers with a common factor: 36 + 8 is the same as 4(9 + 2)

Critical Background Knowledge from Previous Grade Levels

- Apply properties of operations as strategies (3.OA.5)
- Understand and solve the steps of the order of operations without exponents or parentheses (3.OA.8)
- Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations. Represent these problems using equations with a letter standing for the unknown quantity (4.OA.3)

Academic Vocabulary

expression, parentheses, bracket, brace, order of operations, sum, add, multiply, difference

Suggested Models	Suggested Strategies
Eric is playing a video game. At a certain point in the game, he has 31500 points. Then the following events happen, in order: ● He earns 2450 additional points. ● He loses 3310 points. ● The game ends, and his score doubles. Write an expression for the number of points he has at the end of the game. **Which Building Has More Rooms?** There are four office buildings on Pickney Street. The blue building has 22 rooms on each of the 14 floors. Compared to the blue building, the white building has half as many rooms on each floor and half as many floors. Compared to the white building, the red building has double the number of floors and the same number of rooms on each floor. Compared to the blue building, the gray building has twice as many floors and half as many rooms on each floor. Part 1: Write an expression for each building. Do not worry about solving the expressions. Part 2: Write mathematical comparisons that compare each of the following: a) The blue building has _____ as many rooms as the white building. b) The red building has ____ as many rooms as the blue building. c) The red building has ____ as many rooms as the white building. d) The gray building has ___ as many rooms as the blue building. e) The white building has ___ as many rooms as the gray building. f) The red building has ____ as many rooms as the gray building.	• Write numerical expressions when given mathematical expressions in words • Translate numerical expressions into words • Use games such as “I have, who has” with expressions written in numbers and written in words to give students additional opportunities to understand how to read and interpret expressions without evaluating them
Model adapted from: https://www.illustrativemathematics.org/content-standards/5/OA/A/2/tasks/590	

Analyze patterns and relationships (Standard 5.OA.3)

Standard 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Concepts and Skills to Master

- Generate numerical patterns given a set of rules
- Create input/output tables that include an independent variable and two dependent variables
- Form ordered pairs
- Graph data on the coordinate plane
- Describe patterns based on a set of given rules
- Interpret graphs in the first quadrant of the coordinate plane

Teacher Note: In fifth grade students are only expected to work in Quadrant One on a coordinate plane.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols **5.OA.2** Write simple expressions that record calculations **5.G.1** Compose and understand the coordinate plane **5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane	**6.EE.2** Write, read, and evaluate expressions in which letters represent numbers **6.EE.9** Use variables to represent two quantities in real-world problems, write an equation to express one quantity in relation to another **6.EE.7** Solve real-world and mathematical problems by writing and solving equations **6.RP.1** Understand the concept of a ration and use ratio language to describe a ratio relationship between two quantities

Critical Background Knowledge from Previous Grade Levels

- Generate number or shape patterns that follow a given rule (4.OA.5)
- Identify arithmetic patterns and explain them using properties of operations (3.OA.9)

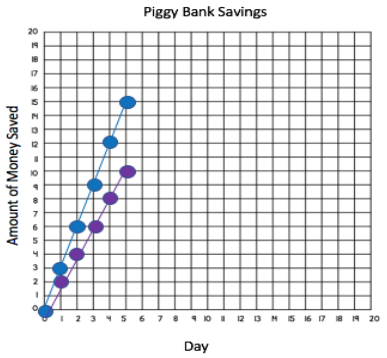
Academic Vocabulary

corresponding terms, coordinate plane, ordered pair, coordinates, pattern, relationship, graph, origin, *x*-axis, *y*-axis, input/output table

Suggested Models

Sam and Taylor both get a new piggy bank to put their earnings into during the summer. Sam earns $2 a day and Taylor earns $3 a day. Create a chart to show how much each child has earned for up to five days. Then plot the points on a coordinate plane to display your data in a line graph and interpret the data.

Days	Sam's Savings	Taylor's Savings
0	$0	$0
1	$2	$3
2	$4	$6
3	$6	$9
4	$8	$12
5	$10	$15



The rule for Sam is add 2 dollars per day. The rule for Taylor is add 3 dollars per day

Suggested Strategies

- Create a table or list displaying data
- Use t-charts to generate the patterns from a given problem with two rules
- Make a table and generate a sequence when given provided rules
- Describe patterns
- Plot ordered pairs on a coordinate grid
- Describe graphs
- Make tables

Understand the place value system (Standards 5.NBT.1–4)

Standard 5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Concepts and Skills to Master

- Know the names and positions of each place value
- Understand the value of each digit in the base 10 system
- Understand that the value of a digit within a number increases or decreases when multiplied or divided by ten in the base ten system
- Accurately multiply multi-digit numbers by powers of 10
- Accurately divide multi-digit numbers by powers of 10
- Model whole numbers and parts of whole numbers with drawings, base ten blocks, and other concrete models

Teacher Note: This is students' first exposure to decimal operations and extends into 5.NBT,2. "Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths. This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is 1/10th the size of the tens place." (http://www.ncpublicschools.org/docs/curriculum/mathematics/scos/5.pdf)

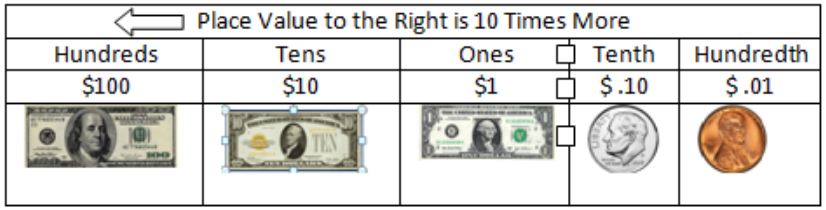
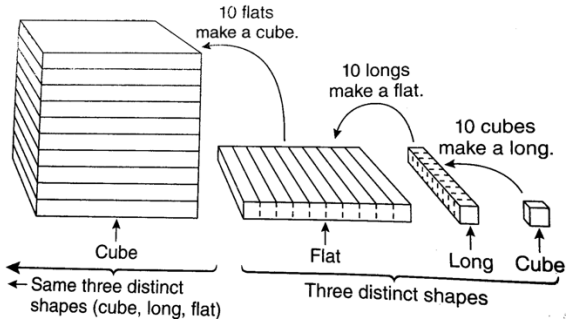
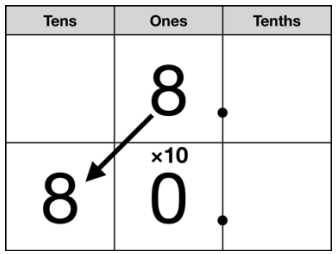
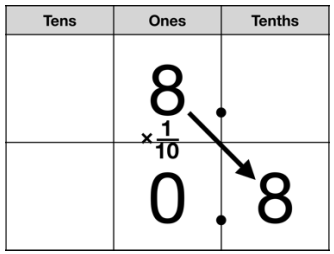
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.3** Read, write, and compare decimals to thousandths. **5.NBT.4** Use place value understanding to round decimals to any place. **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two digit divisors **5.NBT.7** Add, subtract, multiply, and divide decimals to the hundredths **5.MD.1** Convert among different-sized standard measurement units within a given (metric) measurement system	**6.EE.1** Write and evaluate numerical expressions involving whole-number exponents **6.NS.2** Fluently divide multi-digit numbers using the standard algorithm for each operation **6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation

Critical Background Knowledge from Previous Grade Levels

- Recognize that in a multi-digit number, a digit in one place represents 10 times what it represents in the place value to its right (4.NBT.1)
- Multiply one-digit whole numbers by multiples of ten (3.NBT.3)

Academic Vocabulary

base ten system, decimal, names of the place values, tenth, hundredth, thousandth

Suggested Models	Suggested Strategies
   	• Relate money ($1,000, $100, $10, $1, $0.10, $0.01) to the place value of a standard number • Roll dice and place each digit rolled into a place value chart to create the largest, or smallest number possible • Use base ten blocks to represent a given number. Then use blocks to create a number 10 times greater, and 1/10 as large as the original number • Create number line representations of the numbers, including decimal values

Understand the place value system (Standards 5.NBT.1–4)

Standard 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Concepts and Skills to Master

- Understand why multiplying a number by a power of 10 shifts the decimal that many places to the right
- Understand why dividing a number by a power of 10 shifts the decimal that many places to the left
- Understand that when multiplying by powers of 10, the exponent indicates how many places the decimal point is moved to the right in the product, increasing the value 10 times for every decimal place moved
- Understand that when dividing by a power of 10, the exponent indicates how many places the decimal point is moved to the left, decreasing the value of the number by 1/10 for every decimal place moved
- Understand that an exponent indicates the number of times a base is multiplied by itself

Related Standards: Current Grade Level

5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to the left
5.NBT.5 Fluently multiply multi-digit numbers using the standard algorithm
5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths

Related Standards: Future Grade Levels

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents

Critical Background Knowledge from Previous Grade Levels

- Recognize that in a multi-digit number, a digit in one place represents 10 times what it represents in the place value to its right (4.NBT.1)
- Multiply one-digit whole numbers by multiples of ten (3.NBT.3)

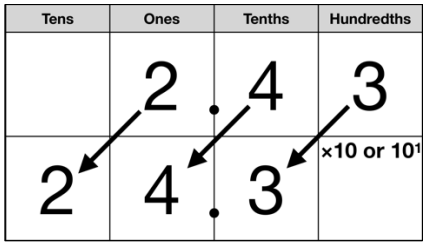
Academic Vocabulary

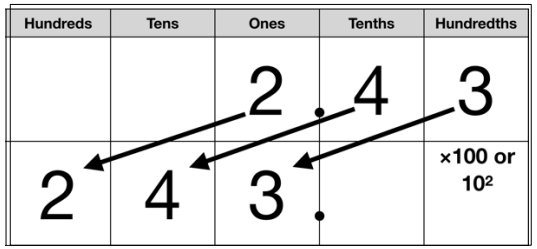
base ten, exponential notation (^), product, power of ten, exponent, base

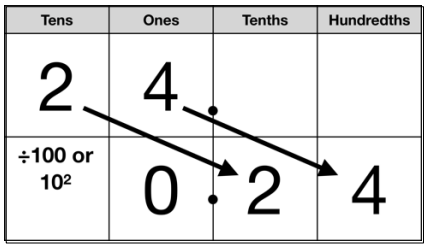
Suggested Models

Multiply Decimals by Powers of Ten

Standard Form	Exponential Form	Examples
10	10^1	$.45 \times 10^1 = 4.5$
100	10^2	$.45 \times 10^2 = 45$
1,000	10^3	$.45 \times 10^3 = 450$







Suggested Strategies

- Display patterns in a number multiplied by powers of ten. Compare the number of zeros in the products in relation to the power of ten factors.
- Use mental math to multiply a factor by multiples of 10, 100, 1000
- Use mental math to divide a dividend by 10, 100, 1000
- Reason about the relative size of a product or quotient based on the power of ten being used to compute.
- Use base ten blocks to model multiplication of division by a power of ten.

Divide Numbers by Powers of Ten

Standard Form	Exponential Form	Examples
10	10^1	$45 \times 10^1 = 4.5$
100	10^2	$45 \times 10^2 = .45$
1,000	10^3	$45 \times 10^3 = .045$

Understand the place value system (Standards 5.NBT.1–4)

Standard 5.NBT.3 Read, write, and compare decimals to thousandths.

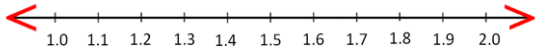
a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. *For example, 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).*

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Concepts and Skills to Master

- Express a given number in multiple ways:
  - base-ten numerals (347.392)
  - number names (three hundred forty-seven and three hundred ninety-two thousandths)
  - expanded form 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)
- Understand that when comparing two numbers, one first looks at the whole number, then the individual digits
- Understand that a number (greater than 0) in the tenths place always has a greater value than the number in the hundredths place
- Generalize that the number with the most tenths is greater
- Understand that if the number of tenths is the same, the number with more hundredths is greater. If the number of tenths and hundredths is the same, the number with more thousandths is greater
- Use terms including greater than, more than, less than, fewer than, equal to, and same as, to describe comparisons
- Use the symbols >, =, and < to correctly to compare decimals to thousandths

Teacher Note: Students compare numbers and record the comparisons with the symbols >, =, and <. Emphasis should be placed on the meaning of quantities rather than tricks such as "the alligator eats the bigger number," etc. The inequality symbols (<, >) are shortcuts for identifying the relationship between two numbers where one is greater or smaller than the other. The statements are read from left to right (e.g., 1.5 < 2.8 is read one and five tenths is less than two and eight tenths) A number line can be used to develop the understanding of the inequality symbols. In fifth grade students are not expected to use the term "inequality" when comparing numbers.



Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left **5.NBT.4** Use place value understanding to round decimals to any place	**6.NS.7** Understand ordering and absolute value of rational numbers. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. **6.EE.8** Write an inequality of the form $x > c$ or $x < c$

Critical Background Knowledge from Previous Grade Levels

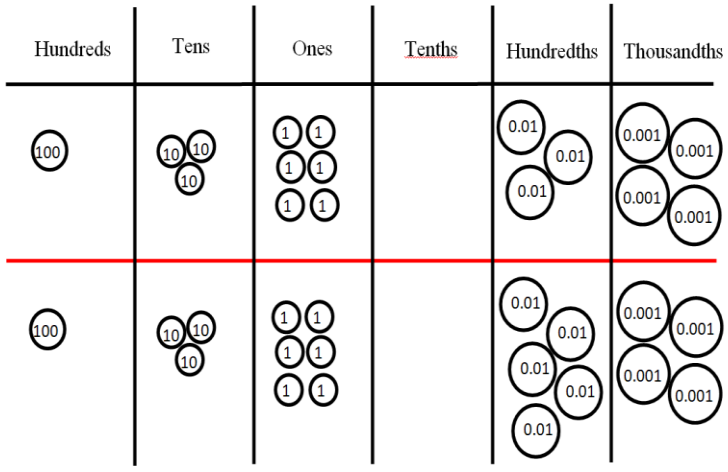
- Compare two decimals to hundredths by reasoning about their size. Record the results of comparisons with the symbols >, <, or = and justify the conclusions (4.NF.7)
- Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons (4.NBT.2)
- Compare two fractions with the same numerator or the same denominator. Record the results of comparisons with the symbols >, =, or < (3.NF.3)

Academic Vocabulary

base-ten numeral (also known as standard form), number names (also known as word form), expanded form, compare, more, fewer, greater than (>), less than (<), equal to (=), same as

Suggested Models

Compare 136.034 and 136.054 using a place value chart.



Each digit in the hundred spot has a value of 100.

Each digit in the ten spot has a value of 30.

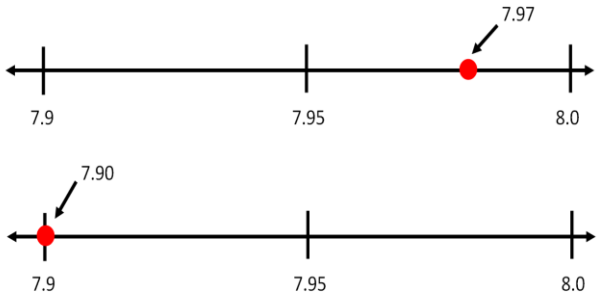
Each digit in the one spot has a value of 6.

Each digit in the tenth spot has a value of 0.

One digit in the hundredth spot has a value of 0.03 and the other has a value of 0.05.

Since 0.05 is a greater value than 0.03, that number has a larger value. So, 136.034 is less than 136.054. Or 136.034 < 136.054.

Compare 7.97 and 7.90 using a double number line.



7.90 is closer to the left of the number line than 7.97. So 7.97 is greater than 7.90. Or 7.97 > 7.90.

Suggested Strategies

- Use concrete materials such as objects on a place value chart, base-ten blocks, and number lines to compare two multi-digit numbers with decimals

Understand the place value system (Standards 5.NBT.1–4)

Standard 5.NBT.4 Use place value understanding to round decimals to any place.

Concepts and Skills to Master

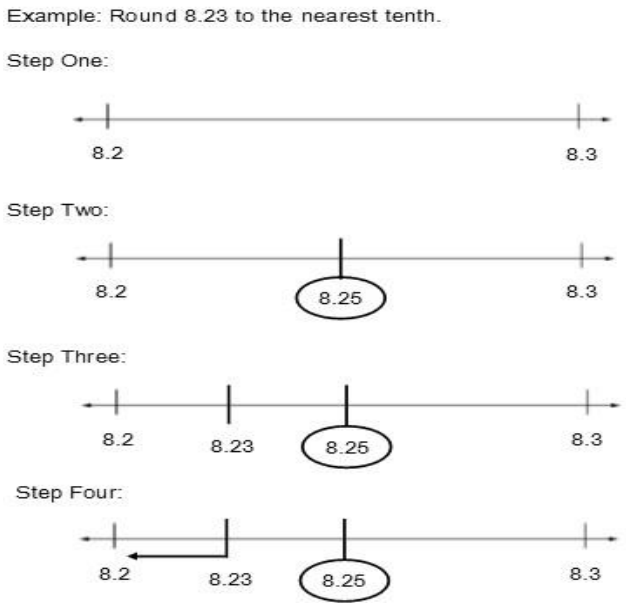
- Use place value understanding to round numbers with decimals to the nearest whole number, tenth, and hundredth
- Understand that rounding can be applied to any place within a number including decimals
- Understand when rounding to the nearest whole number, tenths, or hundredths place, the goal is to approximate the closest number with zero units in the places to the right of the digit to be rounded to (For example, 478.235 rounded to the nearest tenth is 478.2; and 478.235 rounded to the nearest hundredth is 478.24)
- Connect rounding numbers to the location of the number on a number line by identifying the benchmark numbers and using the midpoint to determine which benchmark number is closer (For example, when rounding 478.235 to the nearest tenth, the benchmark numbers are 478.2 and 478.3. The midpoint is 478.25. The number 478.235 is to the left of the midpoint and closer to 478.2 than 478.3. The number 478.235 is therefore rounded to 478.2.)

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.3** Read, write, and compare decimals to thousandths.	Fifth grade is the last grade level in which rounding is specifically addressed. Rounding may be used to support problem solving in various standards in future grade levels.

Critical Background Knowledge from Previous Grade Levels

- Use place value understanding to round multi-digit whole numbers to any place up to 1,000,000 (4.NBT.3)
- Use place value understanding to round two-digit and three-digit numbers to the nearest 10 and 100 (3.NBT.1)

Suggested Models



Academic Vocabulary

round a decimal, benchmark number, midpoint, digits, estimate, close to, nearest place, ones place, tenths place, hundredths place

Suggested Strategies

- Create and use horizontal and vertical open number lines to identify, locate, and label benchmark numbers, midpoints, and target numbers to assist in rounding
- Use base-ten blocks, decimal bars, and drawings to model the concept of rounding with decimals
- Use a place value chart and/or place value disks as a tool for support when rounding
- Use pennies, dimes, and dollars to model rounding

Perform operations with multi-digit whole numbers and with decimals to hundredths (Standards 5.NBT.5–7).

Standard 5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Concepts and Skills to Master

- Extend understanding of multiplication with specified multi-digit numbers to multiply with any multi-digit whole numbers
- Fluently compute products of whole numbers using a variety of strategies including the standard algorithm
- Use properties of operation and place value to explain a standard algorithm
- Understand and explain connections between various multiplication strategies and a standard algorithm

Teacher Note: A standard algorithm of multiplication is neither an expectation nor a focus in fourth grade. Students use multiple strategies for multiplication in grades 3-5. By the end of fourth grade students use a range of algorithms based on place value, properties of operations, and/or the relationships between addition and multiplication to multiply multi-digit whole numbers. Students are expected to fluently multiply multi-digit whole numbers using a standard algorithm by the end of fifth grade. Fifth grade students should not only focus on the standard algorithm, but should progress from strategies used in fourth grade to a standard algorithm.

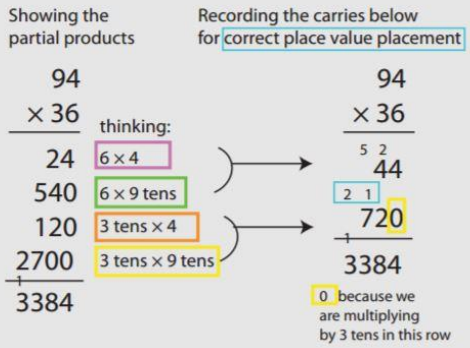
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 **5.NBT.7** Multiply decimals to hundredths, using concrete models or drawings and strategies based on place value **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction (using area models and partial products) **5.MD.5** Relate volume to the operations of multiplication	**6.NS.3** Fluently multiply multi-digit decimals using the standard algorithm **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers

Critical Background Knowledge from Previous Grade Levels

- Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models (4.NBT.5)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (3.OA.3)
- Apply properties of operations as strategies to multiply and divide (3.OA.5)
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90, for example, 9 × 80 and 5 × 60 (3.NBT.3)

Academic Vocabulary

multiply, factor, product, factor pairs, multiples, distributive property, area model, partial products, algorithm

Suggested Models	Suggested Strategies
Methods that compute partial products first  *These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from 30 × 4 = 120 is placed correctly in the hundreds place and the digit 2 from 30 × 90 = 2700 is placed correctly in the thousands place. If these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow box) in the ones place of the second row of the method on the right is there because the whole row of digits is produced by multiplying by 30 (not 3). Colors on the left correspond with the area model above.* *Also see models on the Core Guide for Standard 4.NBT.5	Teacher Note: This standard refers to fluency which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart also using strategies according to the numbers in the problem. This standard builds upon students' work with multiplying numbers in third and fourth grade. In fourth grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding. • Area models • Partial products • Standard algorithm • Compare different models to show how place value is utilized to arrive at the same product
Images Source: http://commoncoretools.me/wp-content/uploads/2015/03/ccss_progression_nbp_k5_2015_03_16.pdf	

Perform operations with multi-digit whole numbers and with decimals to hundredths (Standards 5.NBT.5–7).

Standard 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two- digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Concepts and Skills to Master

- Extend understanding of division with one-digit divisors to divide numbers by two-digit divisors
- Understand how to compute quotients of two-digit divisors and two, three, and four-digit dividends
- Understand how to compute quotients in a variety of situations, including with zeros in various places
- Interpret whole-number quotients of whole numbers with and without remainders from partitive and quotative contexts (Partitive: interpret 560 ÷ 80 as the number of objects in each share when 560 objects are partitioned equally into 80 shares; Quotative: interpret 560 ÷ 80 as a number of shares when 560 objects are partitioned into equal shares of 80 objects each)
- Connect physical representations (objects) to visual representations (drawings)
- Connect physical and visual representations to equations
- Use a variety of strategies to find quotients between the following numbers with and without remainders:
    - two-digit divisors and two-digit dividends
    - two-digit divisors and three-digit dividends
    - two-digit divisors and four-digit dividends

Teacher Note: The standard algorithm of division is neither an expectation nor a focus in fifth grade. Students use multiple strategies for division in grades 3-5. In fourth and fifth grade students use a range of algorithms based on place value, properties of operations, and/or the relationships between subtraction and division to divide multi-digit whole numbers. Students are expected to fluently divide multi-digit whole numbers using the standard algorithm by the end of sixth grade.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.7** Multiply decimals to hundredths, using concrete models or drawings and strategies based on place value	**6.NS.2** Fluently divide multi-digit numbers using the standard algorithm **6.NS.3** Fluently divide multi-digit decimals using the standard algorithm **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers

Critical Background Knowledge from Previous Grade Levels

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models (4.NBT.6)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (3.OA.3)
- Apply properties of operations as strategies to multiply and divide (3.OA.5)

Academic Vocabulary

dividend, divisor, quotient, partial quotients, remainder, place value

Suggested Models	Suggested Strategies
There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?	• Use the relationship between multiplication and division • Use repeated subtraction and sharing as division strategies • Use manipulatives such as base-ten blocks or place-value discs and drawings such as equal groups, arrays, and area models to represent division • Use partial quotients and place value sections to model and visualize division • Explain connections between physical models, visual models, and equations

Student 1

1,716 divided by 16
There are 100 16's in 1,716.
1,716 – 1,600 = 116
I know there are at least 6 16's.
116 - 96 = 20
I can take out at least 1 more 16.
20 - 16 = 4
There were 107 teams with 4 students left over. If we put the extra students on different team, 4 teams will have 17 students.

Student 2

1,716 divided by 16.
There are 100 16's in 1,716.
Ten groups of 16 is 160. That's too big.
Half of that is 80, which is 5 groups.
I know that 2 groups of 16's is 32.
I have 4 students left over.

1716 -1600	100
116 -80	5
36 -32	2
4	

Student 3

1,716 ÷ 16 =
I want to get to 1,716
I know that 100 16's equals 1,600
I know that 5 16's equals 80
1,600 + 80 = 1,680
Two more groups of 16's equals 32, which gets us to 1,712
I am 4 away from 1,716
So we had 100 + 6 + 1 = 107 teams
Those other 4 students can just hang out

Student 4

How many 16's are in 1,716?
We have an area of 1,716. I know that one side of my array is 16 units long. I used 16 as the height. I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16. 100 + 7 = 107 R 4

	100	7
16	100 x 16 = 1,600	7 x 16 =112
	1,716 - 1,600 = 116	116 - 112 = 4

Perform operations with multi-digit whole numbers and with decimals to hundredths (Standards 5.NBT.5–7).

Standard 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. In this standard, dividing decimals is limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Compare the value of the quotient on the basis of the values of the dividend and divisor.

Concepts and Skills to Master

- Use previous understandings for adding and subtracting whole numbers to adding and subtracting decimals to hundredths
- Understand that a whole number can be written with a decimal point followed by one or more zeros
- Understand that when adding or subtracting decimals, units must be aligned with the corresponding places correctly (hundredths are aligned with hundredths; tenths are aligned with tenths; ones are aligned with ones, etc.)
- Use previous understandings for multiplying whole numbers to multiplying decimals to hundredths
- Explain why when multiplying by 0.1 or by 0.01 the product is 10 or 100 times as small as the multiplicand (the digits shift one or two places to the right of the decimal point)
- Use a variety of methods to reason about the placement of a decimal point in the product of two decimals
- Use previous understandings for dividing whole numbers to dividing decimals to hundredths
- Explain why when dividing by 0.1 or by 0.01 the quotient becomes 10 times or 100 times as large as the dividend (the digits shift one or two places to the left of the decimal point)
- Understand that when the decimal point in the divisor is shifted to make a whole number, the decimal point in the dividend should shift the same number of places
- Apply a variety of strategies based on place value to add, subtract, multiply, and divide decimals

Teacher Note: Students are not required to multiply hundredths by hundredths. Expectations for division of decimals is limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Fifth grade students are not required to compute decimal dividends by decimal divisors.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. **5.MD.1** Convert among different-sized standard measurement units within a given measurement system. **5.NF.4, 5.NF.6** Multiply a fraction by a fraction **5.NF.3, 5.NF.7** Divide with fractions	**6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers.

Critical Background Knowledge from Previous Grade Levels

- Fluently multiply and divide within 100 (3.OA.7)
- Fluently add and subtract multi-digit whole numbers using the standard algorithm (4.NBT.4)
- Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers (4.NBT.5)
- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors (4.NBT.6)

Academic Vocabulary
divisor, dividend, whole number, decimal, properties of operations, operation notations

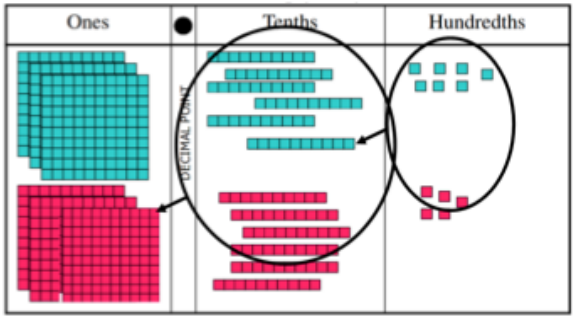
Suggested Strategies
• Use the relationship between addition and subtraction • Use the relationship between multiplication and division • Apply whole number strategies to decimal computation (area model, number line, base-ten blocks)

Suggested Models

Models for Addition and Subtraction With Decimals

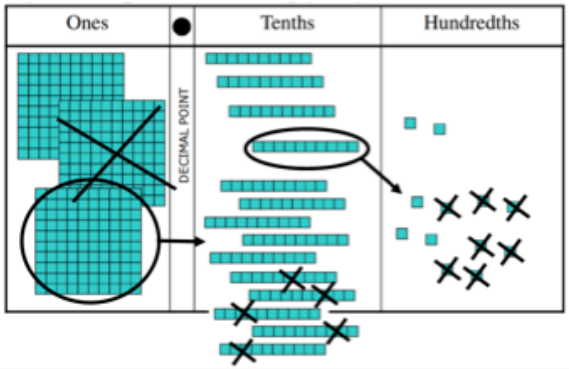
Add using a place value mat

3.57 + 2.65 = 6.22



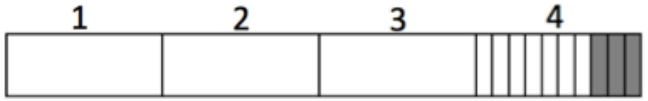
Subtract using a place value mat

3.42 – 1.57 = 1.85



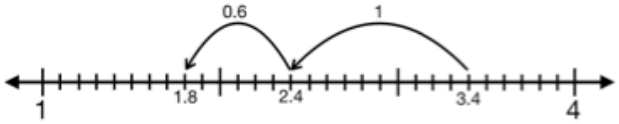
Subtract using a tape diagram

4 – 0.3 = 3.7



Subtract using a number line

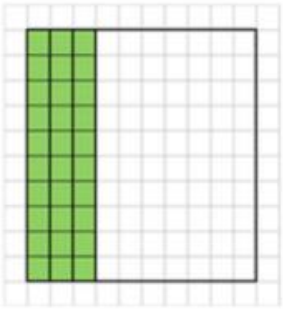
3.4 – 1.6 = 1.8



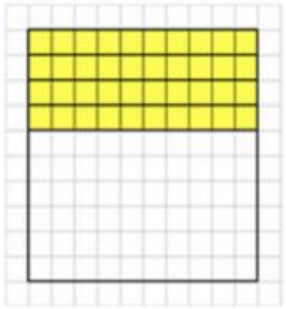
5.NBT.7

Models for Multiplication With Decimals

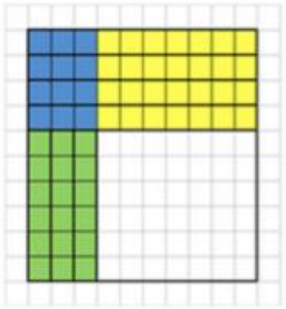
0.3 = 30 hundredths



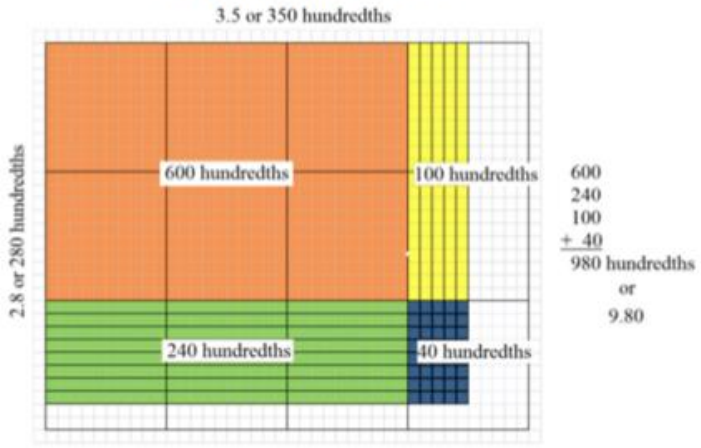
0.4 = 40 hundredths



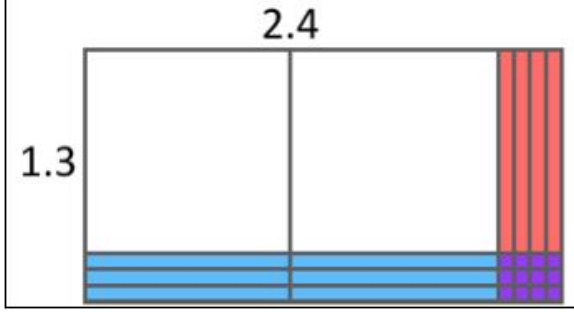
Shaded area is 0.12
or 12 hundredths



Area model for 2.8 x 3.5



Area model for 2.4 x 1.3



Open area model for 2.4 x 1.3

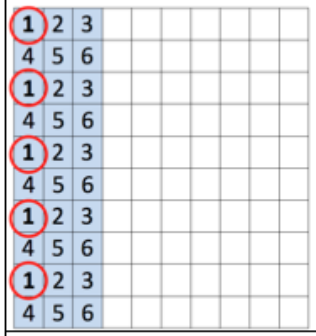
	2	+	.4
1	2		.4
+			
.3	.6		.12

2.4
− 1.3
.12 (0.3×0.4)
.60 (0.3×2)
.40 (1×0.4)
+ 2.00 (1×2)
3.12

5.NBT.7

Models for Division With Decimals

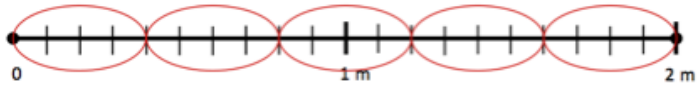
0.30 ÷ 6 = 0.05



0.30 is shaded. The student numbered each of the hundredths 1 through 6 to represent 6 groups. The student then circled the number 1 to show the number of hundredths in each group.

Finding the number of groups

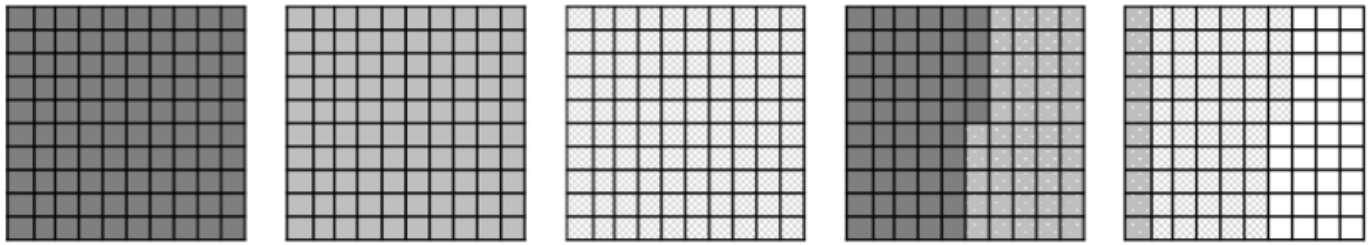
2 ÷ 0.4 = 5



Students could draw a segment to represent 2.0 meters and partition into tenths. They may then circle groups of 4 tenths, determining that there are 5 groups of 4 tenths within 2 meters

Hundred Grid to Model all Operations

A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Make an estimate, find your actual answer, and then compare them.



My estimate is that each runner runs between 1 and 2 miles. If each runner went 2 miles, that would be a total of 6 miles which is too high. If each runner ran 1 mile, that would be 3 miles, which is too low. I used the 5 grids above to represent the 4.65 miles. I am going to use all of the first 4 grids and 65 of the squares in the 5th grid. I have to divide the 4 whole grids and the 65 squares into 3 equal groups. I labeled each of the first 3 grids for each runner, so I know that each team member ran at least 1 mile. I then have 1 whole grid and 65 squares to divide up. Each column represents one-tenth. If I give 5 columns to each runner, that means that each runner has run 1 whole mile and 5 tenths of a mile. Now, I have 15 squares left to divide up. Each runner gets 5 of those squares. So each runner ran 1 mile, 5 tenths and 5 hundredths of a mile. I can write that as 1.55 miles. My answer is 1.55 and my estimate was between 1 and 2 miles. I was pretty close.

Possible solution equations:

4.65 ÷ 3 = 1.55 miles

3 x 1.55 = 4.65 miles

1.55 + 1.55 + 1.55 = 4.65 miles

4.65 – 1.55 – 1.55 = 1.55 miles

Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf; http://www.clayton.edu/portals/636/Content/MATH%203010%20PowerPoints/addition%20and%20subtraction%20decimals.pdf; https://www.ck12.org/book/CK-12-Middle-School-Math-Concepts-Grade-6/section/4.8/

5.NBT.7

Use equivalent fractions as a strategy to add and subtract fractions (Standards 5.NF.1–2).

Standard 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general,* a/b + c/d = (ad + bc)/bd.*)*

Concepts and Skills to Master

- Understand why fractions and mixed numbers must have common denominators to be added or subtracted
- Use visual representations to explain the need for common denominators when adding and subtracting fractions and mixed numbers
- Use multiple strategies to find common denominators to add or subtract fractions including mixed numbers (See strategies below)
- Identify and select efficient strategies to compose and decompose fractions, whole numbers, and mixed numbers flexibly based on the numbers and operations being used in the problem.
- Connect visual models to numerical representations

Teacher Note: It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding or subtracting fractions. Also, not all fractions need to be expressed in lowest terms. Greatest common factor and least common multiple are introduced in Standard 6.NS.4 and are not needed for an understanding of addition and subtraction of fractions.

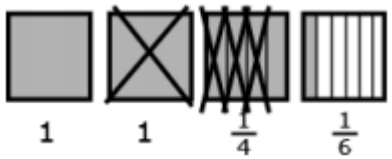
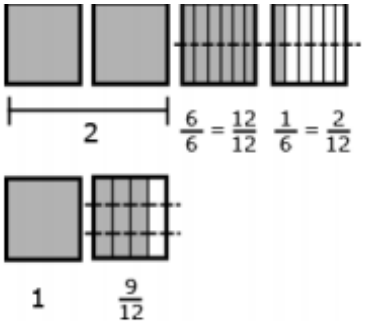
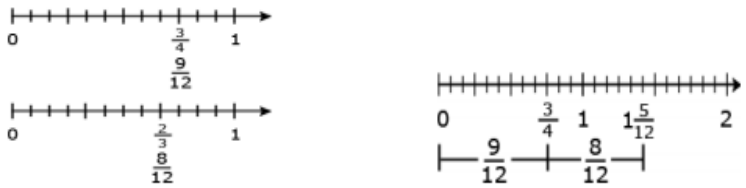
Related Standards: Current Grade Level	Related Standards: Future Grade Level
5.NF.2 Solve real word problems involving addition and subtraction of fractions **5.NBT.7** Add and subtract decimals to hundredths using concrete models or drawings	**6.EE. 7** Solve problems by writing and solving equations of the form x + a = b where variables may be fractions **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions

Critical Background Knowledge from Previous Grade Levels

- Explain why fractions are equivalent by using visual fraction models (4.NF.1)
- Generate equivalent fractions by creating common denominators or numerators (4.NF.2)
- Understand addition and subtraction of fractions as joining and separating parts of the same whole (4 NF 3.a)
- Understand a mixed number is a whole number and a fraction that can also be represented as a fraction greater than 1 (4.NF.3.b)
- Add and subtract fractions with like denominators including mixed numbers (4.NF.3c)

Academic Vocabulary

Common denominator, unlike denominator, like denominator, fraction greater than one, mixed number, numerator, denominator, equivalent fraction, compose, decompose, common multiple

Suggested Models	Suggested Strategies
Example: Using an area model to subtract This model shows $1\frac{3}{4}$ subtracted from $3\frac{1}{6}$ leaving $1+\frac{1}{4}+\frac{1}{6}$. A student can then convert the fractions to $1+\frac{3}{12}+\frac{2}{12}=1\frac{5}{12}$1 + 3/12 + 2/12 = 1 5/12.  $3\frac{1}{6}$ can be expressed as $3\frac{2}{12}$. $3\frac{2}{12}$ can be decomposed to create the problem $2\frac{14}{12}-1\frac{9}{12}=1\frac{5}{12}$. This diagram models a way to show how $3\frac{1}{6}$ and $1\frac{3}{4}$ can be expressed with a denominator of 12 and how $2\frac{14}{12}-1\frac{9}{12}=1\frac{5}{12}$ can be solved.  • Linear model  	Use visual models including number bonds, number lines, fraction strips, tape diagrams, area models, set models, rulers and equations to do the following: • Use equivalent fractions as a strategy to find common denominators in order to add and subtract fractions • Apply understanding of equivalent fractions to rewrite fractions in equivalent forms with common denominators • Use the Multiplicative Identity Property of 1 to transform a fraction into an equivalent fraction and generate equivalent fractions using this principle (Students may, but need not, use the formal term for this property) • Find common denominators through common multiples or finding the product of both denominators

5.NF.1

Use equivalent fractions as a strategy to add and subtract fractions (Standards 5.NF.1–2).

Standard 5.NF.2 Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by, *for example, using visual fraction models or equations to represent the problem*. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize 2/5 + 1/2 = 3/7 as an incorrect result, by observing that 3/7 < 1/2.*

Concepts and Skills to Master

- Understand why fractions and mixed numbers must have common denominators to be added or subtracted
- Use visual representations to explain the need for common denominators when adding and subtracting fractions and mixed numbers
- Use multiple strategies to find common denominators to add or subtract fractions including mixed numbers (See strategies below)
- Identify and select efficient strategies to compose and decompose fractions, whole numbers, and mixed numbers flexibly based on the numbers and operations being used in the problem
- Connect visual models to numerical representations
- Solve real-world problems involving addition and subtraction of fractions, including mixed numbers
- Mentally estimate and assess the reasonableness of an answer

Teacher Note: It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding or subtracting fractions. Also, not all fractions need to be expressed in lowest terms. Greatest common factor and least common multiple are introduced in Standard 6.NS.4 and are not needed for an understanding of addition and subtraction of fractions.

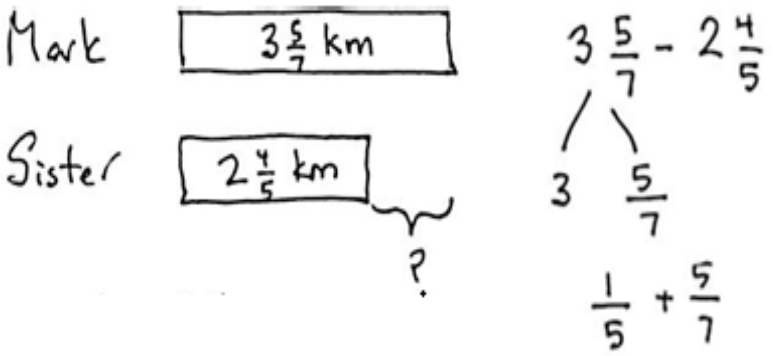
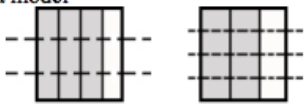
Related Standards: Current Grade Level	Related Standards: Future Grade Level
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) **5.NBT.7** Add and subtract decimals to hundredths using concrete models or drawings	**6.EE. 7** Solve problems by writing and solving equations of the form x + a = b where variables may be fractions **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions

Critical Background Knowledge from Previous Grade Levels

- Explain why fractions are equivalent by using visual fraction models (4.NF.1)
- Generate equivalent fractions by creating common denominators or numerators (4.NF.2)
- Understand addition and subtraction of fractions as joining and separating parts of the same whole (4 NF 3.a)
- Understand a mixed number is a whole number and a fraction that can also be represented as a fraction greater than 1 (4.NF.3.b)
- Add and subtract fractions with like denominators including mixed numbers (4.NF.3c)

Academic Vocabulary

fraction greater than one, mixed number, numerator, denominator, like denominators, unlike denominators, common denominators, equivalent fractions, compose, decompose, common multiple, estimate, reasonableness

Suggested Models	Suggested Strategies
 Example: Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 cup of sugar. How much sugar did he need to make both recipes? • Mental estimation: A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2. • Area model  $\frac{3}{4}$ cup of sugar $\frac{3}{4} = \frac{9}{12}$ $\frac{2}{3}$ cup of sugar $\frac{2}{3} = \frac{8}{12}$ $\frac{3}{4} + \frac{2}{3} = \frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}$	Use visual models including number bonds, number lines, fraction strips, tape diagrams, area models, set models, rulers and equations to do the following: • Use equivalent fractions as a strategy to find common denominators in order to add and subtract fractions • Apply understanding of equivalent fractions to rewrite fractions in equivalent forms with common denominators • Use the Multiplicative Identity Property of 1 to transform a fraction into an equivalent fraction and generate equivalent fractions using this principle (Students may, but need not, use the formal term for this property.) • Find common denominators through common multiples or finding the product of both denominators • Use benchmark fractions (0, $\frac{1}{2}$, 1) to estimate and assess the reasonableness of an answer

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve real-world problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, through the use of visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing three by four, noting that 3/4 multiplied by four equals three, and that when three wholes are shared equally among four people each person has a share of size 3/4. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Concepts and Skills to Master

- Understand that a fraction is a way to represent the division of two quantities (a/b = a÷b)
- Rewrite a whole-number division expression as a fraction. Know that 3/5 "three fifths" can also be interpreted as "3 divided by 5"
- Create story contexts to represent problems involving division of whole numbers to include remainders written as fractions

Related Standards: Current Course	Related Standards: Future Courses
5.NF.4 Multiply a fraction or a whole number by a fraction **5.NF.5** Interpret multiplication as scaling **5.NF.7** Divide whole numbers and unit fractions by each other	**6.RP.2** Understand ratio concepts and ratio reasoning to solve problems **6.G.2** Solve volume problems for solids with unit fraction edge lengths **7.NS.2** Apply and extend operations with fractions to add, subtract, multiply, and divide irrational numbers

Critical Background Knowledge from Previous Grade Levels

- Understand multiplication of a whole number and a fraction as the concept of repeated addition of unit fractions. (4.NF.4)
- Multiply and divide to solve word problems involving whole numbers. (4.OA.2)
- Divide whole numbers by whole numbers. (3.OA.2)

Academic Vocabulary

numerator, denominator, fraction greater than one, mixed number, quotient, divisor, dividend, remainder, fair share, equal shares, sharing, equal size pieces

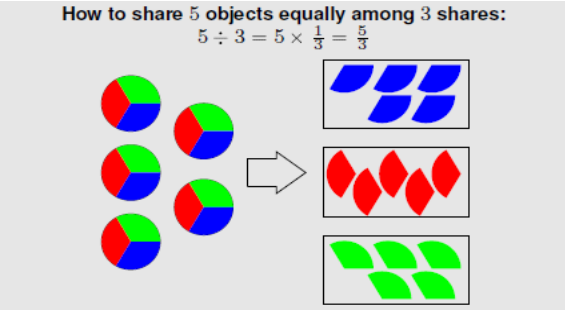
Suggested Models	Suggested Strategies
	• Use concrete and visual fraction models and equations to represent a problem • Convert a division problem into a multiplication problem involving a whole number and unit fraction • Use whole-number multiplication to find the closest whole-number quotient and then partition the remainder into equal groups • Use contexts of word problems to evaluate reasonableness of answers and remainders If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? 9 x 5 = 45 pounds so each person receives 5 pounds with 5 pounds remaining. Partitioning the remaining 5 pounds give each person $\frac{5}{9}$ pounds per person. So each person gets $5\frac{5}{9}$ pounds of rice.

Image Source: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product (*a/b*) x *q* as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations *a* x *q* ÷ *b* using a visual fraction model.

*For example, use a fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (*a/b*) x (*c/d*) = ac/bd.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Concepts and Skills to Master

- Extend multiplying and representing a fraction by a whole number (2 x 1/4 = 1/2) in Grade 4 to multiplying and representing a whole number by a fraction (1/4 x 2 = 1/2).
- Understand that a whole number multiplied by a fraction can be represented as a portion of the whole number (one fourth of 2 is equal to one half)
- Create a story context for an equation of the form (a/b) x q
- Multiply and represent a fraction by a fraction including fractions greater than 1
- Understand that the area of a rectangle is measured in square units and that square units may be fractional units
- Create area models to illustrate the meaning of multiplying fractions and explain the model's relationship to both factors and the product
- Find the area of a rectangle with fractional side lengths by tiling the area with unit squares
- Find that the area of a rectangle with fractional sides is the same as the product of the side lengths

Related Standards: Current Course	Related Standards: Future Courses
5.NBT.7 Perform operations with multi-digit whole numbers and with decimals to the hundredths **5.NF.5b** Apply and extend previous understandings of multiplication and division to multiply and divide fractions	**6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form ax=b for cases in which a, b and x are all non-negative rational numbers **6.G.1 - 4** Solve real-world and mathematical problems involving area, surface area and volume **7.NS.2a** Apply and extend previous understandings of multiplication as an extension from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations **7.SP.3** Draw informal comparative inferences about two populations

Critical Background Knowledge from Previous Grade Levels

- Apply and extend previous understandings of multiplication to multiply a fraction by a whole number (4.NF.4)
- A square with side length one unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (3.MD.5a)
- A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. (3.MD.5 b)
- Geometric measurement: Understand concepts of area and relate area to multiplication and to addition. (3.MD.7c)

Academic Vocabulary

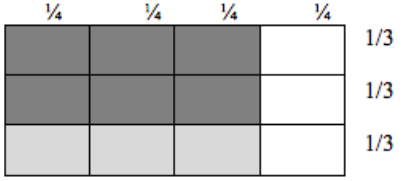
partition, factor, product, numerator, denominator, fraction, whole number, unit Fraction, equivalent, area, length, width, square unit, array, dimension, tiling

ADA Compliant 11/6/2019
USBE ADA Compliant March 2021

Suggested Models

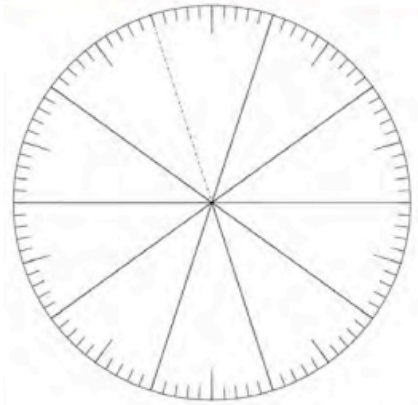
Student 1

I drew a rectangle to represent the whole class. The four columns represent the fourths of a class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is 6/12, which equals 1/2.

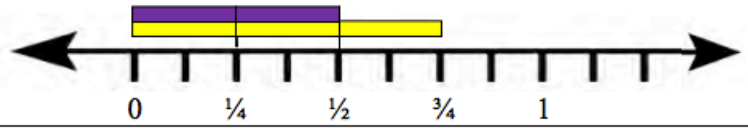


Student 3

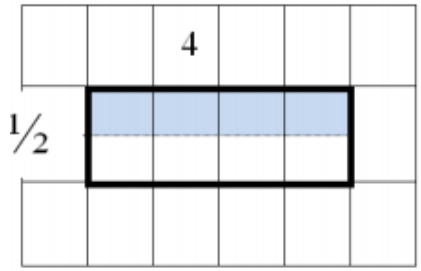
Fraction circle could be used to model student thinking.
First I shade the fraction circle to show the ¾ and then overlay with 2/3 of that?



Student 2

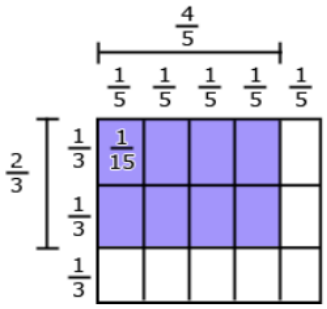


Example: The home builder needs to cover a small storage room floor with carpet. The storage room is 4 meters long and half of a meter wide. How much carpet do you need to cover the floor of the storage room? Use a grid to show your work and explain your answer. In the grid below I shaded the top half of 4 boxes. When I added them together, I added ½ four times, which equals 2. I could also think about this with multiplication ½ x 4 is equal to 4/2 which is equal to 2.



Suggested Strategies

- Create a story context for an equation of the form (a/b) x q (Students create a story problem for 3/5 x 6 such as, • Isabel had 6 feet of wrapping paper. She used 3/5 of the paper to wrap some presents. How much does she have left? • Every day Tim ran 3/5 of mile. How far did he run after 6 days? (Interpreting this as 6 x 3/5)
- Use visual fraction models (area models, tape diagrams, number lines, circles, folding) to multiply a whole number by a fraction
- Create area models to find the area of a rectangle with fractional side lengths and explain the model's relationship to both factors and the product
- Create area models Use graph paper/dot paper to make pictorial representations of area
- Write equations to represent area models showing a product of fractions



The area model and the line segments show that the area is the same quantity as the product of the side lengths.

Image Sources: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf; http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf

5.NF.4

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.5 Interpret multiplication as scaling.

a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, the products of expressions such as 5 x 3 or ½ x 3 can be interpreted in terms of a quantity, three, and a scaling factor, five or ½. Thus in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is five times as big as three, without evaluating the product. Likewise they see ½ x 3 as half the size of three.*

b. Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number (recognizing multiplication by whole numbers greater than one as a familiar case); explain why multiplying a given number by a fraction less than one results in a product smaller than the given number; and relate the principle of fraction equivalence. *For example, 6/10 = (2x3)/(2x5). In general,* a/b = *(*n *x* a*)/(*n *x* b*) has the effect of multiplying* a/b *by one*.

Concepts and Skills to Master

- Understand relationships between the size of factors and products
- Use estimation to check the reasonableness of the products
- Understand multiplication as scaling as expressions that can be interpreted in terms of quantity and scaling factor (5 x 3 is 5 times as big as 3. $\frac{1}{2}$ x 3 is half the size of 3)
- Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number
- Explain why multiplying a given number by a fraction less than one results in a product smaller than the given number
- Understand fraction equivalence

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.OA.2 Write and interpret numerical expressions **5.NF.4b** Find the area of a rectangle with fractional side lengths	**6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b not equal to 0, and use rate language in the context of a ratio relationship **6. RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems.

Critical Background Knowledge from Previous Grade Levels

- Use the four operations to solve word problems. (4.MD.2)
- Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models (4.NF.1)
- Compare two fractions with different numerators and different denominators (4.OA.2)
- Interpret a multiplication equation as a comparison (4.OA.1)
- Interpret products of whole numbers (3.OA.1)
- Interpret whole-number quotients of whole numbers (3.OA.2)

Academic Vocabulary

scaling, array, factor, product, x means “of”, compare, increase, decrease, fraction greater than 1, fraction less than 1, mixed number

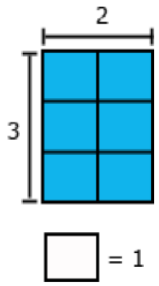
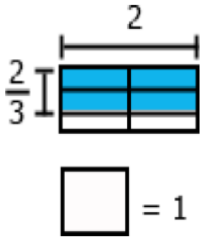
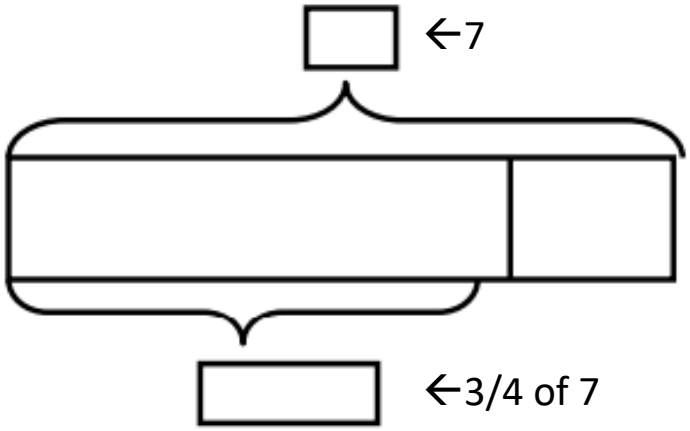
Suggested Models	Suggested Strategies
• Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.  • Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that 2 x 2/3 = 4/3  Example: $\frac{3}{4}$ x 7 is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.  	• Draw models to compare, and reason, about the size of products in relationships to the size of various factors. • Use area models to demonstrate the concept of scaling • Construct viable arguments and critique the reasoning of others about the size of a product compared to the size of one factor on the basis of the size of the other factor. • Use models and/or words to explain why multiplying a given number by a fraction greater than one results in a product greater than the given number • Use models and/or words to explain why multiplying a given number by a fraction less than one results in a product smaller than the given number • Work with multiplying by unit fractions

Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, f*or example, by using visual fraction models or equations to represent the problem*.

Concepts and Skills to Master

- Understand and use various strategies to interpret word problems involving multiplication of fractions and mixed numbers (fraction by a fraction, fraction by a mixed number, mixed number by mixed number)
- Write an equation to represent a word problem and solve the equation

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by an fraction. **5.NF.5** Interpret multiplication as scaling **5.MD.2** Make a line plot to display a data set or measurements in fractions and multiply fractions to solve problems	**6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form ax = b for cases in which a, b and x are all nonnegative rational numbers **6.RP.3** Use ratio and rate reasoning to solve real-world problems **6.G.1 - 4** Solve real-world and mathematical problems involving area, surface area and volume **7.NS.2a** Apply and extend previous understandings of multiplication as an extension from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations

Critical Background Knowledge from Previous Grade Levels

- Apply and extend previous understandings of multiplication to multiply a fraction by a whole number (4.NF.4)
- Use the four operations to solve word problems involving simple fractions (4.MD.2)
- Interpret a multiplication equation as a comparison; Multiply to divide to solve word problems involving multiplicative comparison (4.0A.1, 4.0A.2)
- Interpret products of whole numbers (3.0A.1)

Academic Vocabulary

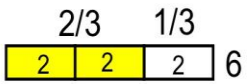
Equation, factors, products, fraction, mixed number

Suggested Models

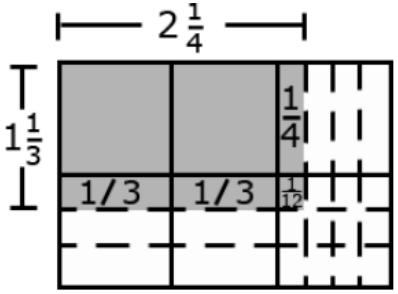
Example: Evan bought 6 roses for his mother. of them were red. How many red roses were there? Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.

$\frac{2}{3}\ x\ 6 = \frac{12}{3} = 4$

4 roses are red



Example: Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft. What will be the area of the school flag?
A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication.



Suggested Strategies

- Use concrete and pictorial area models to represent and make sense of real world problems (unit bars, number lines, area models, linear models, pattern blocks, fraction circles)

Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Use strategies to divide fractions by reasoning about the relationship between multiplication and division. Division of a fraction by a fraction is not a requirement at this grade.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, *for example, by using visual fraction models and equations to represent the problem*. *For example, how much chocolate will each person get if three people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in two cups of raisins?*

Concepts and Skills to Master

- Understand and use visual models to divide a unit fraction by an non-zero whole number (e.g., *⅓ ÷ 4)*
- Understand and use visual models to divide a whole number by a unit fraction. (e.g., *4 ÷ ⅕)*
- Solve real word problems using division of fractions.
- Understand and use the inverse relationship between multiplication and division to reason and solve real world problems.

Teacher Note: This standard is limited to dividing with whole numbers and unit fractions. Fractions divided by fractions will be introduced in 6th grade. This standard should be taught with context and visual models.

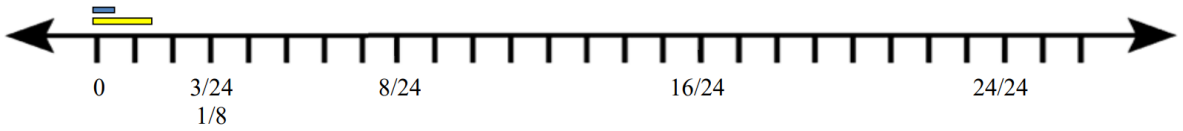
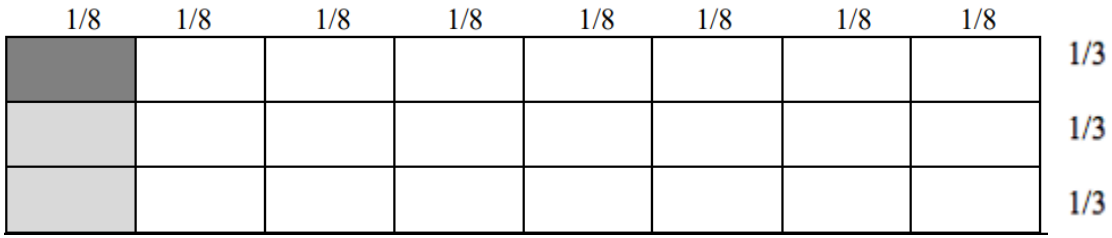
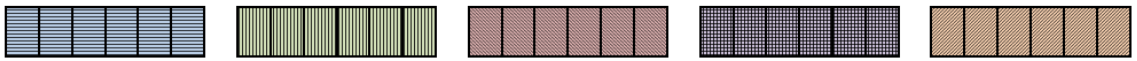
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NF.4 Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction. **5.NF.6** Solve real word-world problems involving multiplication of fractions and mixed numbers. **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**6.NS.1** Interpret and compute quotients of fractions by fractions by applying visual fraction models, equations, and the relationship between multiplication and division. Solve real world problems and explain the meaning of quotients in fraction division problems. **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b ? 0, and use rate language in the context of a ratio relationship.

Critical Background Knowledge from Previous Grade Levels

- Apply and extend previous understanding of multiplication to multiply a fraction by a whole number (4.NF.4)
- Explain why fraction are equivalent by using visual fraction models (4.NF.1)
- Generate equivalent fraction by creating common denominators or numerators (4.NF.2)
- Understand properties of multiplication and the relationship between multiplication and division (3.0A.6)
- Understand that a unit fraction has a numerator of one and a non-zero denominator. (3.NF.1)

Academic Vocabulary

Unit fraction, whole number, quotient, dividend, divisor, equation, inverse operations

Suggested Models	Suggested Strategies
Example: You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get? Expression 1/ 8 ÷ 3  I drew a rectangle and divided it into 8 columns to represent my 1/8. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is 1/24 of the grid or 1/24 of the bag of pens.  Example: Create a story context for 5 ÷ 1/6. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many 1/6 are there in 5? The bowl holds 5 Liters of water. If we use a scoop that holds 1/6 of a Liter, how many scoops will we need in order to fill the entire bowl? I created 5 boxes. Each box represents 1 Liter of water. I then divided each box into sixths to represent the size of the scoop. My answer is the number of small boxes, which is 30. That makes sense since 6 x 5 = 30.  1 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 a whole has 6/6 so five wholes would be 6/6 + 6/6 + 6/6 + 6/6 + 6/6 =30/6	• Use various visual fraction models to illustrate division of a unit fraction by a non-zero whole number and division of a whole number by a unit fraction • Create story context to illustrate division of a unit fraction by a non-zero whole number and division of a whole number by a unit fraction • Solve real-world problems using manipulatives • Give students an equation and have them come up with a real world problem that represents the equation • Relate multiplication of fractions to division of fractions
Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf	

5.NF.7

TABLE 2. Common multiplication and division situations. [1]

	Unknown Product **3 × 6 = ?**	Group Size Unknown *("How many in each group?" Division)* **3 × ? = 18 and 18 ÷ 3 = ?**	Number of Groups Unknown *("How many groups?" Division)* **? × 6 = 18 and 18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example.*** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example.*** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example.*** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2]	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
AREA[3]	What is the area of a 3 cm by 6 cm rectangle?	A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE[4]	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? ***Measurement example.*** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? ***Measurement example.*** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? ***Measurement example.*** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[4] Multiplicative Compare problems appear first in Grade 4, with whole-number values in all places, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs A times as much as the blue hat" results in the same comparison as "A blue hat costs 1/A times as much as the red hat," but has a different subject.

Convert like measurement units within a given measurement system (Standard 5.MD.1).

Standard 5.MD.1 Convert among different-sized standard measurement units within a given measurement system *(for example, convert 5 cm to 0.05 m)*; use these conversions in solving multi-step, real-world problems.

Concepts and Skills to Master

- Convert within a given measurement system expressing smaller units in terms of larger units and larger units in terms of smaller units
- Conceptualize conversions beyond memorized procedures and apply conversions to real-world and multi-step problems

Teacher Note: This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and connection between fractions and decimals rather than teaching mnemonic devices without understanding. Students should not be expected to memorize unit conversions; however, knowing relative sizes of measurement units within systems of units and having repeated exposure to commonly used units will support them in being able express measurements in terms of other units.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm **5.NBT.6** Find whole-number quotients of whole numbers **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value	**6.RP.3d** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities

Critical Background Knowledge from Previous Grade Levels

- Know relative sizes of measurement units within each system of units. Express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table (4.MD.1)
- Use the four operations to solve word problems involving distances, intervals of time, liquid volume, masses of objects, and money (4.MD.2)
- Multiply or divide to solve word problems involving multiplicative comparisons (4.OA.2)
- Solve multi-step word problem posed with whole numbers and having whole number answers using the four operations (4.OA.3)

Academic Vocabulary

All units of measurement in customary and metric systems, including: kilometer, meter, centimeter, millimeter, liter, milliliter, kilogram, gram, milligram, mile, yard, foot, inch, gallon, quart, pint, cup, ton, pound, and ounce as well as abbreviations for symbols (“ = in., ‘ = ft.).

Suggested Models

In fifth grade the main focus is on arriving at the measurements that generate a table. In sixth grade, tables can be discussed in terms of ratios and proportional relationships

Feet	Inches
0	0
	1
	2
	3

Suggested Strategies

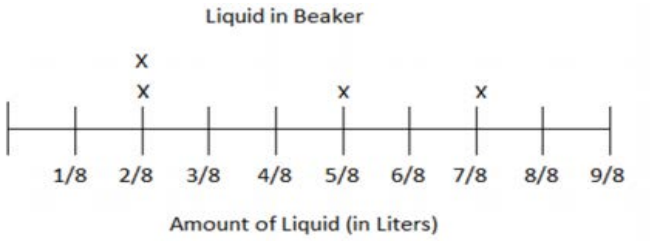
- Explore how the base-ten system supports conversions within the metric system
- Provide a variety of situations for students to measure within a given measurement system and convert those measurements within the same system
- Measure the same object using two different units, then compare the measurements to the size of the units being used
- Draw pictures and models to generalize conversions
- Create a two-column chart or table to notice any patterns for converting within given measurements (see the table to the left)

Image Sources: https://commoncoretools.files.wordpress.com/2012/07/ccss_progression_gm_k5_2012_07_21.pdf, https://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf

Represent and interpret data (Standard 5.MD.2)
Standard 5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, eighths). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given graduated cylinders with different measures of liquid in each, find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally.*
Concepts and Skills to Master
• Make a line plot using provided data sets; include a horizontal scale, title, labels, and straight columns of symbols to represent the data points (• or **X**) • Use a variety of strategies to solve addition and subtraction problems related to data on a line plot Teacher Note: In fifth grade students are not expected to generate measurement data, although they may. Measurement data to be plotted may represent length, volume, or mass. This standard is an extension of the fourth grade standard 4.MD.4.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NF.2 Solve real world problems involving the addition and subtraction of fractions referring to the same whole, including cases of unlike denominators **5.MD.1** Convert among different-sized standards of measurement units within a given measurement system **5.MD.5** Relate volumes to the operations of multiplication, addition and solve real-world and mathematical problems involving volume	**6.SP.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. Choose the most appropriate graph/plot for the data collected

Critical Background Knowledge from Previous Grade Levels
• Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, and eighths). Solve problems involving addition and subtraction with like denominators of fractions by using information presented in line plots (4.MD.4) • Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters (3.MD.4)
Academic Vocabulary
line plot, scale, interval, data set

Suggested Models	Suggested Strategies
	• Analyze and interpret data on created line plots • Measure objects to one-eighth of a unit, including length, mass, and liquid volume • Given a created line plot, write and solve questions related to the data
Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf	

Understand concepts of geometric measurement and volume, as well as how multiplication and addition relate to volume (Standard 5.MD.3-5)

Standard 5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
a. A cube with side length one unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

Concepts and Skills to Master

- Understand that volume is an attribute of a solid or three dimensional figure
- Understand volume is measured in cubic units
- Explain why figures should be packed without gaps or overlaps
- Understand packing as a way to measure volume in cubic units

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.MD.4 Measure volumes to the operations of multiplication and addition and solve real-world and mathematical problems in volume **5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world mathematical problems	**6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems **7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects

Critical Background Knowledge from Previous Grade Levels

- Recognize area as an attribute of plane figures and understand concepts of area measurement (3.MD.5)

Academic Vocabulary

Cube, Unit cube (n^3), One cubic unit, Volume, Solid figure, Overlapping (a partial face to partial face creates a gap) vs. stacking (full face to full face), packing

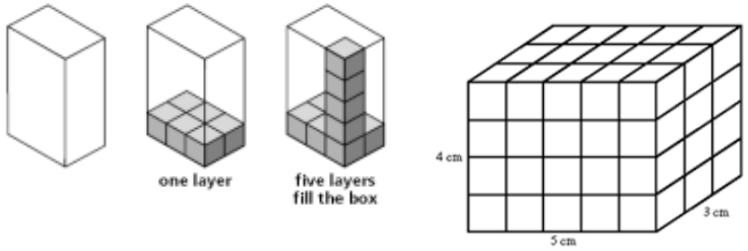
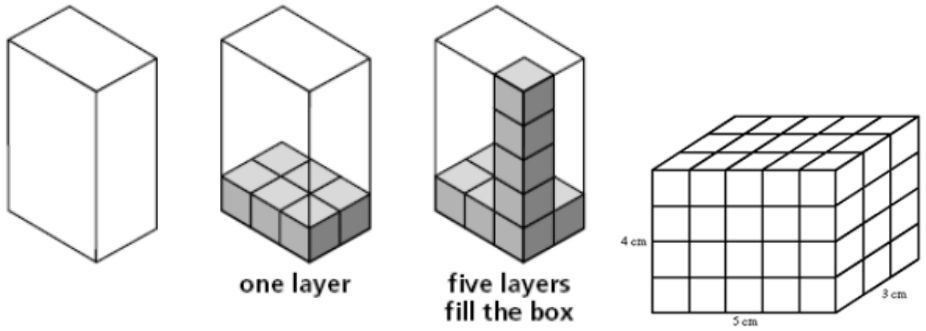
Suggested Models	Suggested Strategies
	• Fill a rectangular container with unit cubes and then with non-unit objects (marbles, packing peanuts, pom poms, etc.) to show how to represent volume • Explore the concept of volume as an extension from area with the idea that students are covering an area (the bottom of the rectangular prism) with a layer of unit cubes and then adding layers of unit cubes on top of the bottom layer

Image Sources: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf; https://commoncoretools.files.wordpress.com/2012/07/ccss_progression_gm_k5_2012_07_21.pdf

Understand concepts of geometric measurement and volume, as well as how multiplication and addition relate to volume (Standard 5.MD.3-5)	
Standard 5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.	
Concepts and Skills to Master	
• Pack cubes into right rectangular prisms and count the cubes to determine the volume • Use the appropriate unit to measure volume while counting cubes	
Related Standards: Current Course	Related Standards: Future Courses
5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement **5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world mathematical problems	**6.G.2** Find the volume of a right rectangular prism with appropriate fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by volumes of right rectangular prisms with fractional edge lengths **7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects
Critical Background Knowledge	
• Recognize volume as an attribute of solid figures and understand concepts of volume measurement (5.MD.3) • A solid figure can be packed using unit cubes is said to have a volume of *n* cubic units (5.MD.3b)	
Academic Vocabulary	
Cubic in., Cubic ft., Cubic cm., Improvised units (non-standard cubic units)	
Suggested Models	Suggested Strategies
	• Explore the concept of volume as an extension from area with the idea that students are covering an area (the bottom of the rectangular prism) with a layer of unit cubes and then adding layers of unit cubes on top of the bottom layer • Given a specified amount of cubes with several factors (24, 36, etc.), make as many rectangular prisms as possible with a volume of the specified cubic units recording possible dimensions • Build solid figures with unit cubes/linking cubes and determine the volume
Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf	

Understand concepts of geometric measurement and volume, as well as how multiplication and addition relate to volume (Standard 5.MD.3-5)

Standard 5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, *for example, to represent the associative property of multiplication*.

b. Apply the formulas *V = l* x *w* x *h and V = b* x *h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Concepts and Skills to Master

- Discover that multiplying the three edge lengths of a rectangular prism results in finding the volume
- Explain how to relate counting cubes to the formula for finding volume
- Understand and apply the formulas $V = l \times w \times h$ *and* $V = b \times h$
- Understand and solve real-world situations and problems by recognizing that volume is the number of cubic units needed to fill a solid figure
- Decompose a composite shape into rectangular prisms and Understand that volume is additive, the volumes of two or more solid figures added together is the volume of the composite figure

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement **5.MD.4** Measure volumes to the operations of multiplication and addition and solve real-world and mathematical problems in volume	**6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems **7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects

Critical Background Knowledge from Previous Grade Levels

- Apply the area and perimeter formulas for rectangles (4.MD.3)
- Measure and estimate liquid volumes (3.MD.2)
- Measure area by counting unit squares and relate area to the operations of multiplication and addition (3.MD.6, 3.MD.7)
- Compose three-dimensional shapes (1.G.2b)

Academic Vocabulary

right rectangular prism, base, area of base (*b*), length (*l*), height (*h*), width (*w*), volume *(V*), formula, additive

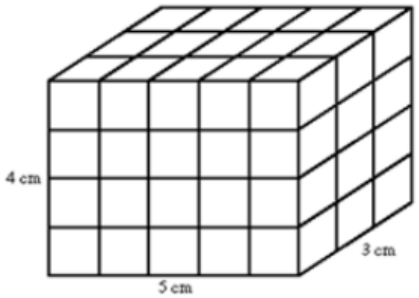
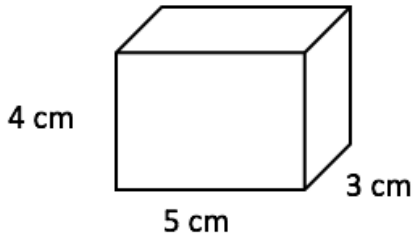
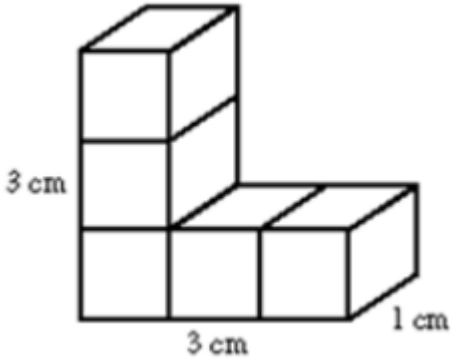
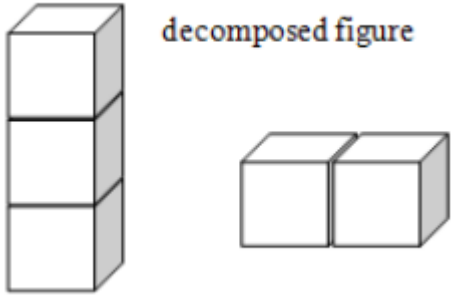
Suggested Models	Suggested Strategies
   	• Pack a right rectangular prism with cubes and use equations to represent the model • Solve problems involving more than one right rectangular prism by building with cubes and decomposing the shape • Solve problems to find the dimensions when given the total volume • Solve problems when given a pre-determined number of cubes to make as many right rectangular prisms possible with that volume • Build prisms in layers, determine base layer and use multiplication to calculate volume

Image Sources: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf, https://commoncoretools.files.wordpress.com/2012/07/ccss_progression_gm_k5_2012_07_21.pdf

Graph points on the coordinate plane to solve real-world and mathematical problems in quadrant one (Standards 5.G.1–2).

Standard 5.G.1 Compose and understand the coordinate plane.

a. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.

b. Using quadrant one on the coordinate plane, understand that the first number in a coordinate pair indicates how far to travel from the origin in the direction of the horizontal axis, and the second number indicates how far to travel in the direction of the vertical axis, with the convention that the names of the two axes and the coordinates correspond (*x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

Concepts and Skills to Master

- Compose the coordinate plane
- Describe the coordinate plane using mathematically correct language, including the terms *x*-axis, *y*-axis, origin
- Understand that the origin represents 0 on the *x*-axis and 0 on the *y*-axis
- Understand that an ordered pair describes a location with respect to the origin
- Understand that ordered pairs are written as (*x*, *y*), with *x* being the distance from the origin in the horizontal direction and y being the distance from the origin in the vertical direction
- Name points using ordered pairs of whole numbers
- Locate points given an ordered pair of whole numbers

Teacher Note: Students at this grade level are required to work in Quadrant I only.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.G.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane **5.OA.3** Form ordered pairs consisting of corresponding terms from two numerical patterns and graph the ordered pairs on a coordinate plane	**6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates **6.NS.7** Understand ordering and absolute value of rational numbers **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane

Critical Background Knowledge from Previous Grade Levels

- Draw perpendicular and parallel lines (4.G.1)
- Represent fractions on a number line diagram (3.NF.2)
- Represent whole numbers on a number line diagram (2.MD.6)

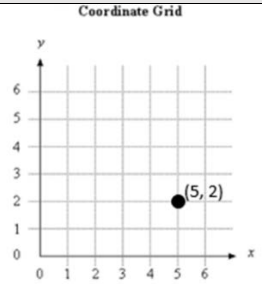
Academic Vocabulary

perpendicular, right angle, intersect, vertical, horizontal, axis, *x*-axis, *y*-axis, coordinate plane/grid, origin, *x*-coordinate, *y*-coordinate, ordered pair, intervals, coordinates, Quadrant I

Suggested Strategies

- Locate points on horizontal and vertical number lines
- Tape axes on a tiled area and have students stand in the correct location given an ordered pair
- Play coordinate grid Battleship (adaptation from regular Battleship)

Suggested Models



Graph points on the coordinate plane to solve real-world and mathematical problems in quadrant one (Standards 5.G.1–2)

Standard 5.G.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Concepts and Skills to Master

- Understand that ordered pairs are written as (x, y), with x being the distance from the origin in the horizontal direction and y being the distance from the origin in the vertical direction
- Name points using ordered pairs of whole numbers
- Locate points given an ordered pair of whole numbers
- Identify real-world situations that could be represented on a coordinate plane
- Interpret the value of the x- and y-coordinates within a given situation

Teacher Note: Students at this grade level are required to work in Quadrant I only.

Related Standards: Current Course	Related Standards: Future Courses
5.G.1 Compose and understand the coordinate plane **5.OA.3** Form ordered pairs consisting of corresponding terms from two numerical patterns and graph the ordered pairs on a coordinate plane	**6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates **6.NS.7** Understand ordering and absolute value of rational numbers **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane **6.EE.9** Analyze the relationship between the dependent and independent variables using graphs and tables **7.RP.3d** Explain what a point (*x*, *y*) on a graph means

Critical Background Knowledge from Previous Grade Levels

- Compose and understand the coordinate plane (5.G.1)
- Draw perpendicular and parallel lines (4.G.1)
- Represent fractions on a number line diagram (3.NF.2)
- Represent whole numbers on a number line diagram (2.MD.6)
- Partition rectangles into rows and columns of equal sized squares (2.G.2)

Academic Vocabulary

perpendicular, intersect, vertical, horizontal, *x*-axis, *y*-axis, coordinate plane/grid, origin, *x*-coordinate, *y*-coordinate, ordered pair, Quadrant I, coordinates

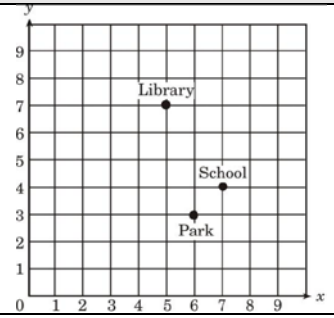
Suggested Models	Suggested Strategies
	• Create a treasure map on a coordinate grid. Give clues and locations using ordered pairs to find a treasure • Tape axes on a tiled area and have students stand in the correct location given an ordered pair • Play coordinate grid Battleship (adaptation from regular Battleship) • Use maps with identified locations. State the coordinates of various buildings or points of interest • Identify the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms. • Present students with graphs that have labeled axes (outside temperature and number of ice cream treats sold) and given a point, ask them to determine what the value of the x- or the y-coordinate represents

Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf

Classify two-dimensional figures into categories based on their properties. (Standards 5.G.3–4).

Standard 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and all squares are rectangles, so all squares have four right angles.*

Concepts and Skills to Master

- Identify, compare, contrast, and describe the attributes of two-dimensional figures
- Recognize shapes that belong to the larger category; Identify examples and non-examples of two-dimensional figures
- Understand that the larger category includes other subcategories. For example, conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides

Teacher Note: Note that in the U.S., that the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. Both definitions are accepted in the United States. Utah has adopted the inclusive definition. **A trapezoid is a quadrilateral with at least one pair of parallel sides.** The inclusive definition is the most accepted definition worldwide and is the definition used by the Utah State Board of Education for standard and assessment purposes. The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do not appear until middle school.

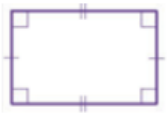
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.G.4 Classify two-dimensional figures in a hierarchy based on properties.	**6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes. **7.G.2** Draw geometric shapes with given conditions.

Critical Background Knowledge from Previous Grade Levels

- Understand that shapes in different categories may share attributes, and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (4.G.1)
- Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (4.G.2)
- Recognize angles as geometric figures that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. (4.MD.5)
- Understand that shapes in different categories may share attributes, and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (3.G.1)
- Recognize and draw shapes having specified attributes, such as a given number of sides of angles. Identify and describe quadrilaterals, squares, rectangles, and trapezoids (2.G.1)

Academic Vocabulary

polygon, angle, line, parallel, perpendicular, triangle, quadrilateral, pentagon, trapezoid, hexagon, parallelogram, rectangle, rhombus, square, acute angle, right angle, obtuse angle, two-dimensional, subset, subcategories, properties, line segment

Suggested Models	Suggested Strategies
Here is an example when a parallelogram is a rectangle:  Here is an example when a parallelogram is *not* a rectangle: 	• Decide whether each of these statements is always, sometimes, or never true. If it is sometimes true, draw and describe a figure for which the statement is true and another figure for which the statement is not true. (See suggested model.) - A rhombus is a square - A triangle is a parallelogram - A square is a parallelogram - A square is a rhombus - A parallelogram is a rectangle - A trapezoid is a quadrilateral • Lead discussions having students reason about the attributes of shapes
Image Source: https://www.illustrativemathematics.org/content-standards/5/G/B/3/tasks/1941	

Classify two-dimensional figures into categories based on their properties. (Standards 5.G.3–4)

Standard 5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

Concepts and Skills to Master

- Reason about the attributes of two-dimensional shapes by examining
- Classify two-dimensional figures in a hierarchy based on properties
- Relate certain categories of shapes as categories of other categories

Teacher Note: Note that in the U.S., that the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. Both definitions are accepted in the United States. Utah has adopted the inclusive definition. **A trapezoid is a quadrilateral with at least one pair of parallel sides.** The inclusive definition is the most accepted definition worldwide and is the definition used by the Utah State Board of Education for standard and assessment purposes.

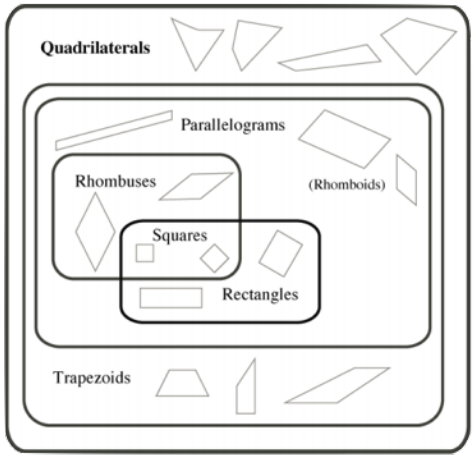
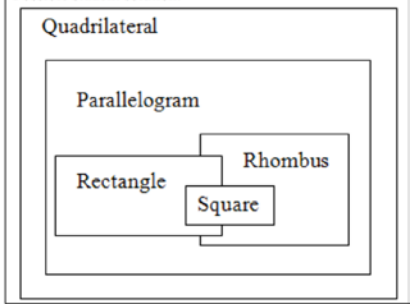
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category	**6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes. **7.G.2** Draw geometric shapes with given conditions.

Critical Background Knowledge from Previous Grade Levels

- Understand that shapes in different categories may share attributes, and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (4.G.1)
- Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (4.G.2)
- Recognize angles as geometric figures that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. (4.MD.5)
- Understand that shapes in different categories may share attributes, and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (3.G.1)
- Recognize and draw shapes having specified attributes, such as a given number of sides of angles. Identify and describe quadrilaterals, squares, rectangles, and trapezoids (2.G.1)

Academic Vocabulary

polygon, angle (∠), line, parallel (∥),perpendicular (⊥), triangle, quadrilateral, pentagon, hexagon, parallelogram, rectangle, rhombus, square, acute angle, right angle (), obtuse angle, trapezoid, two-dimensional, subcategory, category

Suggested Models	Suggested Strategies
Venn diagram showing classification of quadrilaterals  Note that rhomboids are parallelograms that are not rhombuses or rectangles. This example uses the inclusive definition of trapezoid (see p. [pageref "T(E)"]). quadrilateral – a four-sided polygon. parallelogram – a quadrilateral with two pairs of parallel and congruent sides. rectangle – a quadrilateral with two pairs of congruent, parallel sides and four right angles. rhombus – a parallelogram with all four sides equal in length. square – a parallelogram with four congruent sides and four right angles. Possible student solution:  	• Sort given shapes using a graphic organizer such as a bull's-eye graph or Venn diagram or reference chart • Use graphic organizers, diagrams, reference charts • Sequence shapes and their properties into a hierarchy • Lead discussions having students reason about the attributes of shapes • Create a property lists for a given two-dimensional figure (for example quadrilaterals) Assign students to work with one type of quadrilateral. List as many properties as they can that apply to their shape. Compare and contrast the given shapes and their properties • Make a property list using headings such as sides, angles, symmetries etc. Students can add shapes to the given property categories
Image Sources: http://commoncoretools.me/wp-content/uploads/2014/12/ccss_progression_gk6_2014_12_27.pdf; http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf	