Generalize place value understanding for multi-digit whole numbers by analyzing patterns, writing whole numbers in a variety of ways, making comparisons, and rounding (Standards 4.NBT.1–3)

Standard 4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

Concepts and Skills to Master

- Understand the places of numbers and the value of each place
- Model place and value relationships showing how a digit in one place represents ten times what it represents in the place to its right (Use manipulatives such as place value blocks, mats, discs, etc.)
- Understand that the value of each place is ten times greater than the place to the right
- Understand that the value of each place is ten times less than the place to the left
- Multiply and divide numbers by multiples of tens, hundreds, thousands, etc. to one million (For example: 70 × 100 = 7,000 5,000 × 10 = 50,000 and 700 ÷ 70 = 10 50,000 ÷50 = 1,000)

Teacher Note: This standard is a prerequisite to 5.NBT.1 and 5.NBT.2, where students will describe the shifting of digits when multiplying and dividing numbers by multiples of ten.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
4.OA.1–2 Interpret a multiplication equation as a comparison **4.NBT.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form **4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place **4.NBT.4** Fluently add and subtract multi-digit whole numbers **4.NBT.5** Multiply a whole number up to four digits by a one-digit whole number, and multiply two two-digit numbers using strategies based on place value **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value	**5.NBT.1** Recognize that in a multi-digit number, a digit in one place represent 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2** Explain patterns in the numbers of zeros of the product when multiplying a number by powers of 10

Critical Background Knowledge from Previous Grade Levels

- Multiply one-digit whole numbers by multiples of ten (3.NBT.3)
- Represent and solve problems involving multiplication and division within 100 (3.OA.1–4, 7)
- Understand the relationship between multiplication and division and fluently multiply and divide within 100 (3.OA.5–6)
- Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (2.NBT.1)

Academic Vocabulary

inverse operation, base ten numeral (standard form), value, place, and place value, digit, multiply, divide

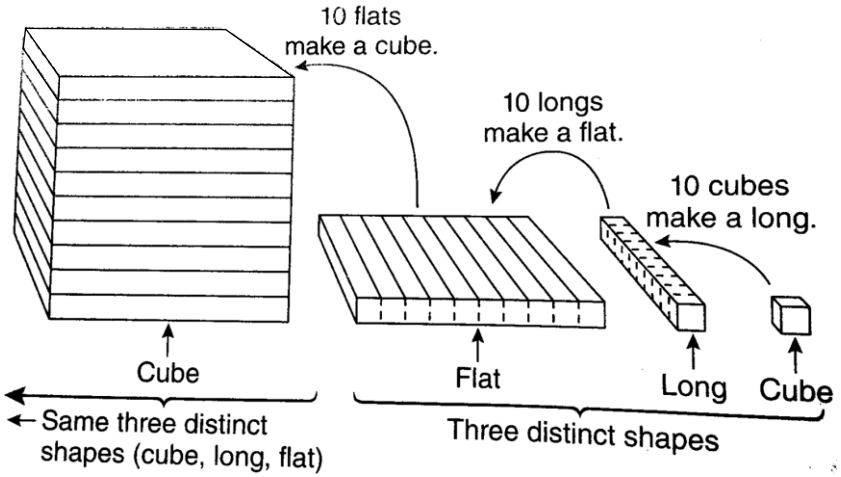
Suggested Models	Suggested Strategies
Each digit in the number 78 becomes one hundred times as much as its original value. The 8 ones becomes 8 hundreds. The 7 tens becomes 7 thousands. **Thousands** | **Hundreds** | **Tens** | **Ones** 7 | 8 | 7 | 8 ×100 7 | 8 | 0 | 0	• Explore patterns that involve moving digits to different places in a given numeral • Investigate patterns associated with the answers obtained to problems such as the following: 7 × 10 7 × 100 7 × 1,000 7 × 10,000 Relate the findings to patterns on the place value chart and using concrete models as shown below.  With every three places, the shapes repeat.

Image Source: A., V. D., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2014). *Teaching student-centered mathematics* Grades 3-5. Boston: Pearson.

Generalize place value understanding for multi-digit whole numbers by analyzing patterns, writing whole numbers in a variety of ways, making comparisons, and rounding (Standards 4.NBT.1–3)

Standard 4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Concepts and Skills to Master

- Express a given number in multiple ways:
  - base-ten numerals (42,371)
  - base-ten word form (4 ten thousands, 2 thousands, 3 hundreds, 7 tens, and 1 one)
  - number names (forty-two thousand, three hundred seventy-one)
  - expanded form (40,000 + 2,000 + 300 + 70 + 1)
- Understand that when comparing two numbers, one looks at the whole number, not just individual digits
- Understand the role of commas when reading a whole number
- Understand that a number (greater than 0) in the thousands place always has a greater value than the number in the hundreds place
- Line up numbers by place value and describe the place value of each digit to compare the numbers
- Understand that if the number of thousands is the same, the number with more hundreds is greater. If the number of thousands and hundreds is the same, the number with more tens is greater
- Use terms including greater than, more than, less than, fewer than, equal to, and same as, to describe comparisons
- Use the symbols >, =, and < to correctly compare multi-digit numbers

Teacher Notes: Emphasis should be placed on the meaning of quantities rather than tricks such as “the alligator eats the bigger number.” The inequality symbols (<, >) are shortcuts for identifying the relationship between two numbers where one is greater or smaller than the other. The statements are read from left to right (for example, 15,000 < 28,000 is read fifteen thousand is less than twenty-eight thousand).

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. **4.NF.7** Compare two decimals to hundredths by reasoning about their size. Record the results of comparisons with the symbols >, <, or = and justify the conclusions.	**5.NBT.3** Read, write, and compare decimals to thousandths. **6.NS.7** Understand ordering and absolute value of rational numbers. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. **6.EE.8** Write an inequality of the form $x > c$ or $x < c$

Critical Background Knowledge from Previous Grade Levels

- Compare two fractions with the same numerator or the same denominator. Record the results of comparisons with the symbols >, +, or < (3.NF.3)
- Read and write numbers to 1,000 using base-ten numerals, number names and expanded form (2.NBT.3)
- Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (2.NBT.4)
- Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (2.NBT.1)

Academic Vocabulary

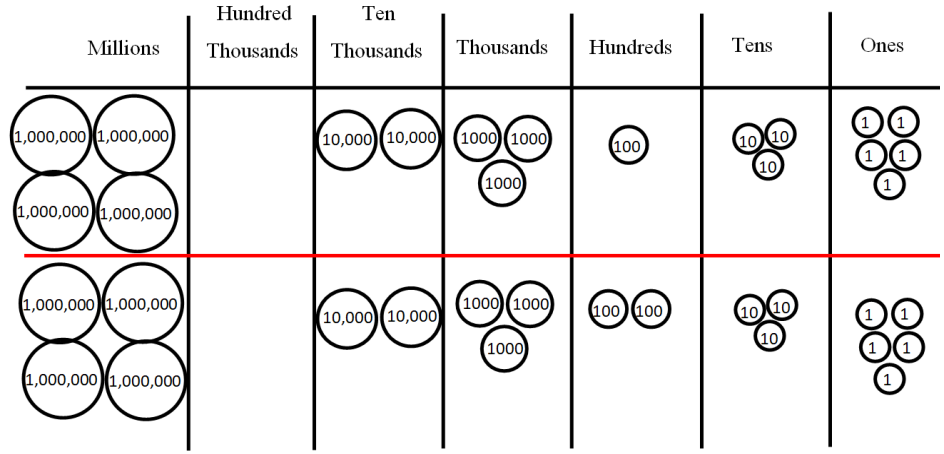
base-ten numeral (formally known as standard form), number names (formally known as word form), expanded form, compare, more, fewer, greater than (>), less than (<), equal to (=), same as

Suggested Strategies

- Use concrete materials such as objects on a place value chart, base-ten blocks, and number lines to compare two multi-digit numbers

Suggested Models

Compare 4,023,135 and 4,023,235 using a place value chart.



Each digit in the million spot has a value of 4,000,000.

Each digit in the hundred thousand spot has a value of 0.

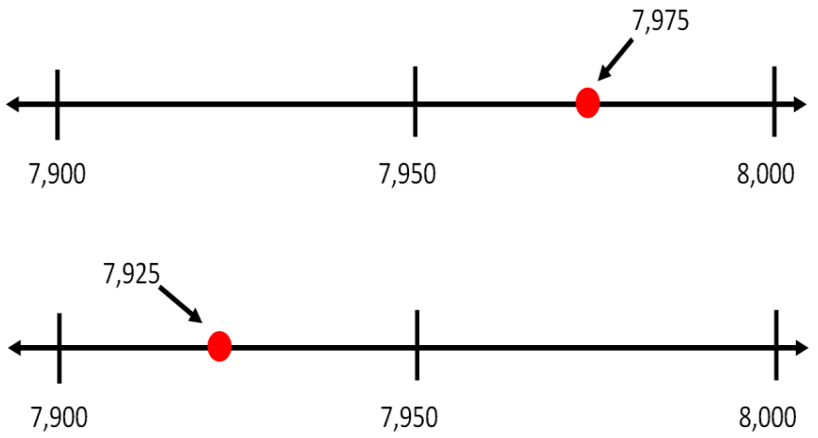
Each digit in the ten thousand spot has a value of 20,000.

Each digit in the thousand spot has a value of 3,000.

One digit in the hundred spot has a value of 100 and the other has a value of 200.

Since 200 is a greater value than 100, that number has a larger value. So, 4,023,135 is less than 4,023,235. Or 4,023,135 < 4,023,235.

Compare 7,975 and 7,925 using a double number line.



7,925 is closer to the left of the number line than 7,975. So 7,975 is greater than 7,925. Or 7,975 > 7,925.

Generalize place value understanding for multi-digit whole numbers by analyzing patterns, writing whole numbers in a variety of ways, making comparisons, and rounding (Standards 4.NBT.1–3)

Standard 4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

Concepts and Skills to Master

- Use place value understanding to round whole numbers less than or equal to 1,000,000
- Understand that rounding can be applied to any place within a number
- Understand when rounding to the nearest tens, hundreds, thousands, ten-thousands, hundred-thousands, or millions place, the goal is to approximate the closest number with zero units in the places to the right of the digit to be rounded to (For example, 478,235 rounded to the nearest ten-thousand is 480,000; and 478,235 rounded to the nearest hundred-thousand is 500,000)
- Connect rounding numbers to the location of the number on a number line by identifying the benchmark numbers and using the midpoint to determine which benchmark number is closer (For example, when rounding 478,235 to the nearest ten-thousand, the benchmark numbers are 470,000 and 480,000, the midpoint is 475,000. The number 478,235 is to the right of the midpoint and closer to 480,000 than 470,000. The number 478,235 is therefore rounded to 480,000. See the model below)

Teacher Note: Rounding to the unit represented by the place farthest to the left is typically easier for students and often sufficient for practical purposes. Rounding to the unit represented by a place in the middle of a number may be more difficult for students as the surrounding digits can be distracting. For example, it may be easier for a student to round 478,235 to 500,000 rather than to 480,000. Students should have experience rounding multi-digit numbers to various places.

Related Standards: Current Course

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right

4.OA.3 Solve multi-step word problems and assess the reasonableness of answers using mental computation and estimation strategies including rounding

Related Standards: Future Courses

5.NBT.4 Use place value understanding to round decimals to hundredths

Critical Background Knowledge from Previous Grade Levels

- Use place value understanding to round two-digit and three-digit numbers to the nearest 10 and 100 (3.NBT.1)
- Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form (2.NBT.3)
- Understand that the three-digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand the value of each digit in three-digit numbers (2.NBT.1)

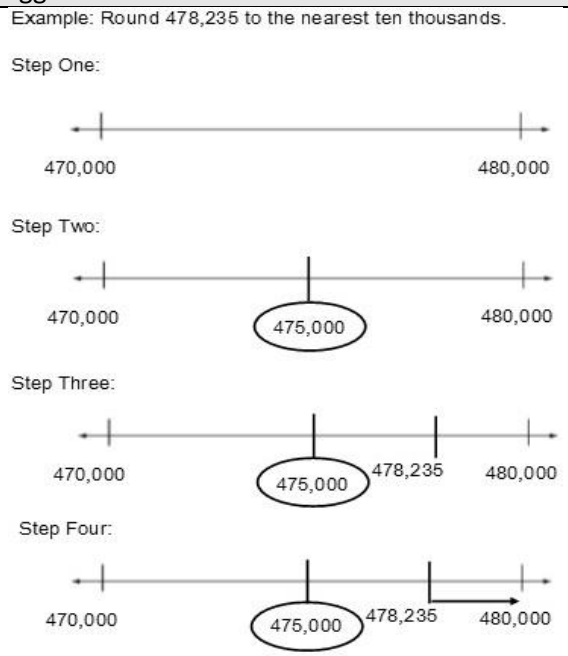
Academic Vocabulary

round, benchmark number, midpoint, digit, estimate, close to, nearest place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place

Suggested Strategies

- Create and use horizontal and vertical open number lines to identify, locate, and label benchmark numbers, midpoints, and target numbers to assist in rounding
- Use base-ten blocks to model rounding up to the thousands place
- Use a place value chart and/or place value disks as a tool for support when rounding
- While songs and mnemonic stories may be engaging, they should not be used in place of developing conceptual understanding of rounding. If these are to be used, they should come after conceptual understanding has been developed
- Use drawings to model the concept of rounding

Suggested Model



Use place value understanding and properties of operations to perform multi digit addition, subtraction, multiplication, and division using a one-digit divisor (Standards 4.NBT.4–6)

Standard 4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Concepts and Skills to Master

- Extend understanding of addition and subtraction of multi-digit whole numbers
- Fluently compute sums and differences of whole numbers using a variety of strategies including the standard algorithm
- Use properties of operation and place value to explain the standard algorithm
- Build understanding and explain connections between various addition and subtraction strategies and the standard algorithm

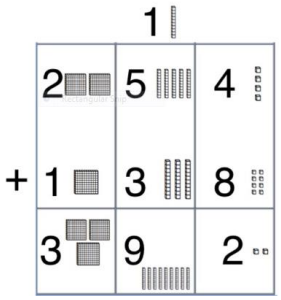
Teacher Note: The standard algorithms of addition and subtraction are neither an expectation nor a focus in second grade. Students use multiple strategies for addition and subtraction in grades K-3. By the end of third grade students use a range of algorithms based on place value, properties of operations, and/or the relationships between addition and subtraction to add and subtract multi-digit whole numbers. Students are expected to fluently add and subtract multi-digit whole numbers using the standard algorithm by the end of fourth grade. Fourth grade students should not only focus on the standard algorithm, but should progress from strategies used in grades K-3 to the standard algorithm. "The standards define a computation algorithm as a set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. The Standards do not specify a particular standard algorithm for each operation." http://commoncoretools.me/wp-content/uploads/2015/03/ccss_progression_nbp_k5_2015_03_16.pdf

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right	**5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm **5.NBT.6** Find quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies **6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm

Critical Background Knowledge from Previous Grade Levels

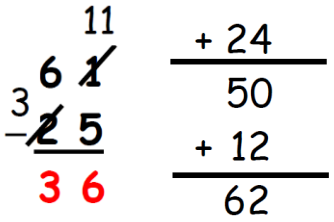
- Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (3.NBT.2)
- Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (2.NBT.5)
- Add and subtract within 1,000 using concrete models or drawings (2.NBT.7)

Suggested Models



Place Value Blocks

203 (-4)= 199
- 46 (-4)=- 42
157
Compensation



38
+ 24
50
+ 12
62

Academic Vocabulary

sum, difference, total, addends

Suggested Strategies

- Use base ten models and connect the model to the algorithm
- Connect standard algorithms to strategies for addition and subtraction

Image Source: DSD Advantage Math 4.NBT

Use place value understanding and properties of operations to perform multi digit addition, subtraction, multiplication, and division using a one-digit divisor (Standards 4.NBT.4–6)

Standard 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Concepts and Skills to Master

- Extend understanding of multiplication with one-digit numbers to multiply specified multi-digit numbers
- Understand how to compute products of one-digit numbers and multiples of 10, 100, and 1,000
- Use the distributive property to decompose numbers into multiples of 10, 100, and 1,000 and multiply those multiples by one-digit numbers to solve for products
- Explain the pattern when multiplying by a value of 10, 100, or 1,000
- Demonstrate understanding of the relationships between pictures and/or equations representing multiplying whole numbers
- Use a variety of strategies to multiply the following numbers:
  - one-digit number by a one-digit number
  - one-digit number by a two-digit number
  - one-digit number by a three-digit number
  - one-digit number by a four-digit number
  - two-digit number by a two-digit number

Teacher Note: A standard algorithm of multiplication is neither an expectation nor a focus in fourth grade. Students use multiple strategies for multiplication in grades 3-5. By the end of fourth grade students use a range of algorithms based on place value, properties of operations, and/or the relationships between addition and multiplication to multiply multi-digit whole numbers. Students are expected to fluently multiply multi-digit whole numbers using a standard algorithm by the end of fifth grade.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison **4.OA.3** Solve multi-step word problems posed with whole numbers and having whole-number answers using multiplication **4.OA.4** Find factor pairs and recognize multiples **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors **4.MD.2** Use the four operations to solve measurement word problems **4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems	**5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm **5.NBT.7** Multiply decimals to hundredths, using concrete models or drawings and strategies based on place value **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction (using area models and partial products) **6.NS.3** Fluently multiply multi-digit decimals using the standard algorithm

Critical Background Knowledge from Previous Grade Levels

- Interpret the products of whole numbers, such as interpreting 5 × 7 as the total number of objects in 5 groups of 7 objects each (3.OA.1)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (3.OA.3)
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers (3.OA.4)
- Apply properties of operations as strategies to multiply and divide (3.OA.5)
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. By the end of third grade, know from memory all products of two one-digit numbers (3.OA.7)
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90, for example, 9 × 80 and 5 × 60 (3.NBT.3)
- Relate area to the operations of multiplication and addition (3.MD.7)
- Use addition to find the total number of objects arranged in rectangular arrays. Partition a rectangle into rows and column of same-sized squares and count to find the total number of squares (2.OA.4, 2.G.2)

Academic Vocabulary

equal groups, array, area model, multiply, factor, product, factor pairs, multiples, distributive property, partial products, multiples of 10, 100, and 1,000

Suggested Strategies

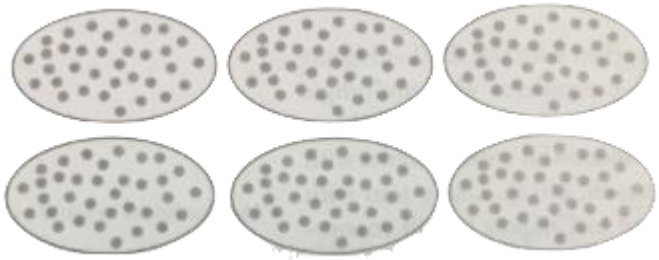
- Use objects (base-ten blocks or place-value discs) and drawings (equal groups, arrays, and area models) to represent multiplication
- Write partial product equations to represent arrays and area models; Explain connections between physical/visual models and equations
- Use the distributive property to solve multiplication problems
- Apply the commutative or associative properties of multiplication

Suggested Models

Teacher Note: These models are ordered in a progression from most concrete to more abstract and more efficient. While it may be acceptable to begin with individual objects to connect to third grade strategies, students should progress towards more efficient strategies.

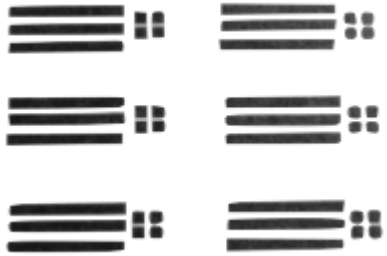
Equal groups with groupable objects for 6 × 34 = 204:

34 + 34 + 34 + 34 + 34 + 34 = 204



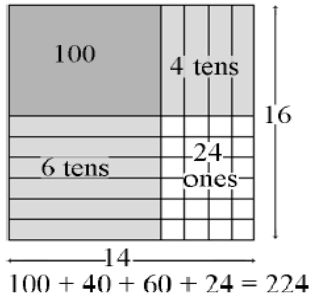
Equal groups with pre-grouped base-ten objects for 6 × 34 = 204:

34 + 34 + 34 + 34 + 34 + 34 = 204



Array with base-ten blocks for 6 × 34 = 204:

Area model with base-ten blocks or graph paper for 16 × 14 = 224:

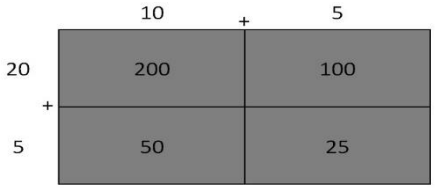


100 + 40 + 60 + 24 = 224

Area model for 6 × 3,253:

18,000 + 1,200 + 300 + 18 = 19,518

Area model for 25 × 15:



300 + 75 = 375

Partial products for 25 × 15:

25
× 15
25 (5 × 5)
100 (5 × 20)
50 (10 × 5)
+ 200 (10 × 20)
375

Distributive Property for 8 x 347:
(300 × 8) + (40 × 8) + (7 × 8) =
2,400 + 320 + 56 = 2,776

Distributive Property for 25 × 15:
(20 × 10) + (5 × 10) + (20 × 5) + (5 × 5) =
200 + 50 + 100 + 25 = 375

Partial products for 8 x 347:
347
× 8
2,400 (300 × 8)
320 (40 × 8)
+ 56 (7 × 8)
2,776

Image Source: Van de Walle, John A. (2014). Teaching student-centered mathematics. Developmentally appropriate instruction for grades 3-5. Boston :Pearson pp.180

Use place value understanding and properties of operations to perform multi digit addition, subtraction, multiplication, and division using a one-digit divisor (Standards 4.NBT.4–6)

Standard 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Concepts and Skills to Master

- Extend understanding of division within 100 to divide specified multi-digit numbers by one-digit divisors
- Use a variety of strategies to find quotients between the following numbers with and without remainders:
    - one-digit divisors and one-digit dividends
    - one-digit divisors and two-digit dividends
    - one-digit divisors and three-digit dividends
    - one-digit divisors and four-digit dividends
- Compute quotients in a variety of situations, including with zeros in various places
- Interpret whole-number quotients of whole numbers with and without remainders from partitive and quotative contexts (Partitive: interpret 560 ÷ 8 as the number of objects in each share when 560 objects are partitioned equally into eight shares; Quotative: interpret 560 ÷ 8 as a number of shares when 560 objects are partitioned into equal shares of eight objects each)
- Demonstrate understanding of the relationships between concrete models, pictures, and/or equations
- Understand remainders as the quantity remaining when the divisor does not divide equally into the dividend
- Interpret remainders in relation to standard 4.OA.3

Teacher Note: A standard algorithm of division is neither an expectation nor a focus in fourth grade. There is not just one standard algorithm and students should use multiple strategies for division in grades 3-5. By the end of fourth grade students use a range of algorithms based on place value, properties of operations, and/or the relationships between subtraction and division to divide multi-digit whole numbers. Students are expected to fluently divide multi-digit whole numbers using a standard algorithm by the end of sixth grade.

Related Standards: Current Course	Related Standards: Future Courses
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison **4.OA.3** Solve multi-step word problems posed with whole numbers using the four operations, including problems in which remainders must be interpreted **4.OA.4** Find factor pairs and recognize multiples **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right **4.MD.2** Use the four operations to solve measurement word problems	**5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors **5.NBT.7** Divide decimals to hundredths **5.NF.3** Interpret a fraction as division of the numerator by the denominator **6.NS.2** Fluently divide multi-digit numbers using the standard algorithm **6.NS.3** Fluently divide multi-digit decimals using the standard algorithm

Critical Background Knowledge from Previous Grade Levels

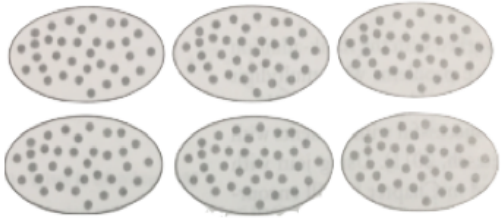
- Interpret whole-number quotients of whole numbers. For example, interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into eight shares (partitive), or as a number of shares when 56 objects are partitioned into equal shares of eight objects each (quotative) (3.OA.2)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (3.OA.3)
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers (3.OA.4)
- Apply properties of operations as strategies to multiply and divide (3.OA.5)
- Understand division as an unknown-factor problem. Understand the relationship between multiplication and division (multiplication and division are inverse operations). For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8 (3.OA.6)
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. By the end of third grade, know from memory all products of two one-digit numbers (3.OA.7)
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90, for example, 9 × 80 and 5 × 60 (3.NBT.3)
- Use addition to find the total number of objects arranged in rectangular arrays. Partition a rectangle into rows and columns of same-sized squares and count to find the total number of squares (2.OA.4, 2.G.2)

Academic Vocabulary

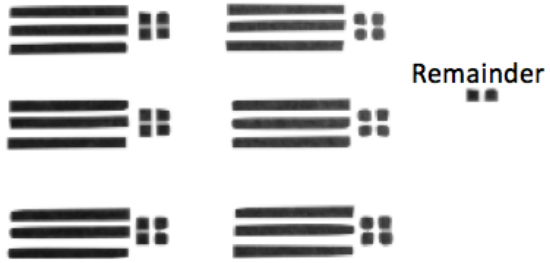
dividend, divisor, quotient, equal groups, partial quotients, remainder, place value

Suggested Models

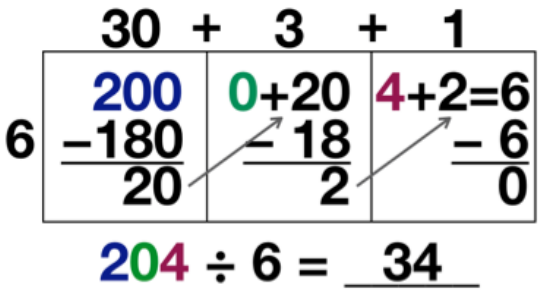
Equal groups with individual objects for $206 \div 6 = 34\ R\ 2$



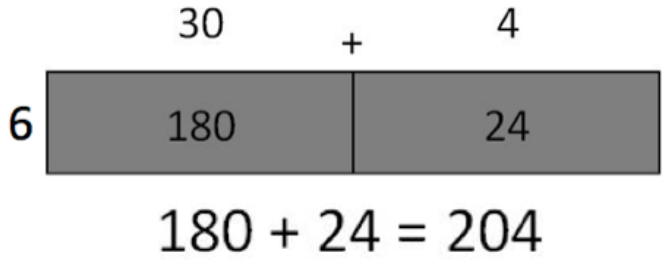
Equal groups with base-ten objects for $206 \div 6 = 34\ R\ 2$



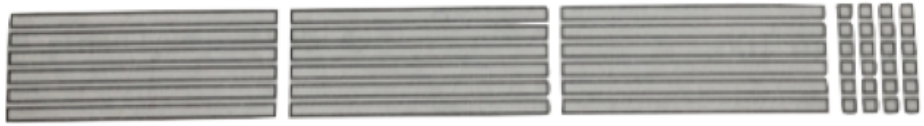
Connect multiplication to division, area model for $204 \div 6 = 34$



Connect multiplication to division, area model for $204 \div 6 = 34$



Array with base-ten blocks for $204 \div 6 = 34$



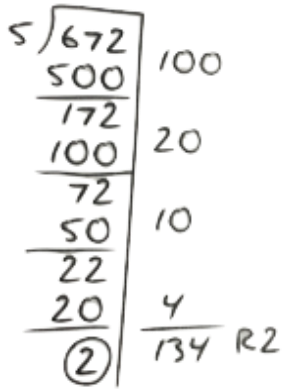
How many groups of 5 are in 672? (At least 100)
Use 100 as the first partial quotient. 100 × 5 = 500
Subtract 672 – 500 = 172

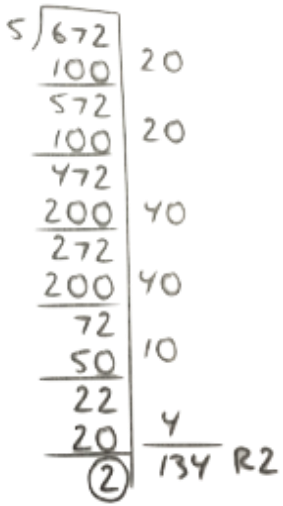
How many groups of 5 are in 172? (At least 20)
Use 20 as the second partial quotient. 20 × 5 = 100
Subtract 172 – 100 = 72

How many groups of 5 are in 72? (At least 10)
Use 10 as the third partial quotient. 10 × 5 = 50
Subtract 72 – 50 = 22

How many groups of 5 are in 22? (At least 4)
Use 4 as the fourth partial quotient. 4 × 5 = 20
Subtract 22 – 20 = 2

Add the partial quotients and record any remainders.
100 + 20 + 10 + 4 = 134
Answer: 134 R2





Suggested Strategies

- Use the relationship between multiplication and division
- Use repeated subtraction and sharing as division strategies
- Use manipulatives such as base-ten blocks or place-value discs and drawings such as equal groups, arrays, and area models to represent division
- Use area models and partial quotients to model, explain, and visualize division
- Explain connections between concrete models, pictures, and/or equations

Images Source: Van de Walle, John A. (2014). Teaching student-centered mathematics. Developmentally appropriate instruction for grades 3-5. Boston :Pearson pp.180, 189, 190