MATHEMATICS

UTAH CORE **GUIDES** GRADE **3**

UTAH STATE BOARD OF EDUCATION 250 EAST 500 SOUTH P.O. BOX 144200 SALT LAKE CITY, UTAH 84114-4200 SYDNEE DICKSON, Ed.D., STATE SUPERINTENDENT OF PUBLIC INSTRUCTION

USBE ADA Compliant March 2021

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.1 Interpret products of whole numbers, such as interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

Concepts and Skills to Master

- Understand multiplication as combining equal groups of objects
- Model skip counting on a number line
- Understand that in a multiplication equation, the first factor equals the number of groups and the second factor equals the number in each group
- Find the total number of objects within equal groups (5 × 7 = 35; 5 groups of 7 is 35)
- Write multiplication expressions and equations to represent pictures
- Draw pictures to represent multiplication expressions and equations

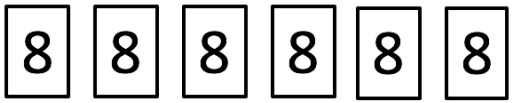
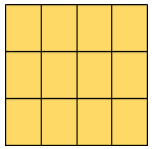
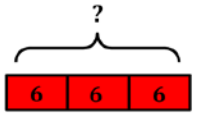
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.2 Interpret whole-number quotients of whole numbers **3.OA.3** Use multiplication and division to solve word problems involving equal groups, arrays, and measurement quantities **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand both division as an unknown-factor problem and the relationship between multiplication and division **3.OA.7** Fluently multiply and divide within 100	**4.OA.1, 4.OA.2** Interpret and solve a multiplication equation as a comparison **4.NBT.5** Multiply multi-digit whole numbers **5.NBT.5** Fluently multiply multi-digit whole numbers **4.NF.4, 5.NF.4** Apply and extend previous understandings of multiplication to fractions

Critical Background Knowledge from Previous Grade Levels

- Use addition to find the total number of objects in an array (2.OA.4)
- Skip count by fives and tens (2.NBT.2)

Academic Vocabulary

equal groups, array, multiplication, factor, product, equation

Suggested Models	Suggested Strategies
 Write an equation that can help you find the total number of points on the stars. 3 × 5 = 15  Frank bought six boxes of crayons. Each box of crayons has 8 crayons in it. How many crayons does he have?    	• Model equal groups with various counters • Discuss real-life situations where objects are in groups • Use and compare number lines, bar models, and area models

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.2 Interpret whole-number quotients of whole numbers. *For example, interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into eight shares (partitive), or as a number of shares when 56 objects are partitioned into equal shares of eight objects each (quotative).*

Concepts and Skills to Master

- Understand that division may represent two different situations: partitive (fair sharing) and quotative (measurement)
- Understand division as repeated subtraction to find the number of equal groups
- Find how many equal groups can be made from a certain number of objects
- Find how many objects can be shared equally among a certain number of groups
- Solve and interpret division problems
- Model a division equations using pictures, objects, or numbers
- Use objects and drawings to represent equal groups
- Use objects, drawings, expressions, and equations to represent division situations

Teacher Note: This standard focuses on two distinct models of division: partitive and quotative. Partitive or fair share models provide students with the total number of objects and the number of groups. Students must solve for the number in each group. Quotative or measurement models provide students with the total number of objects and the number of objects in each group. Students must solve for the number of groups. Students are not expected to know or produce the terms partitive and quotative but should be exposed to them.

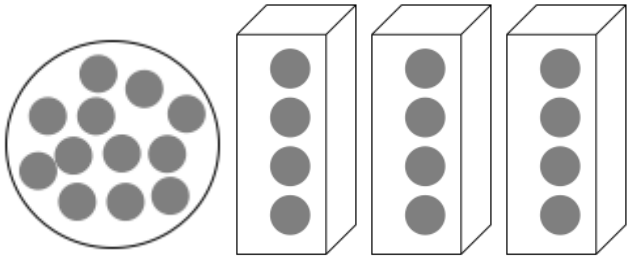
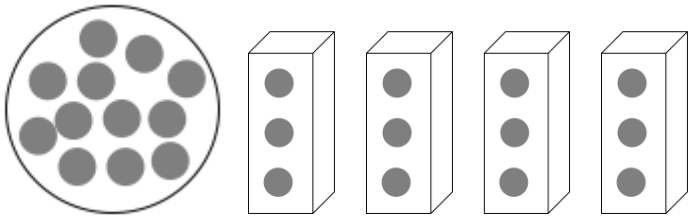
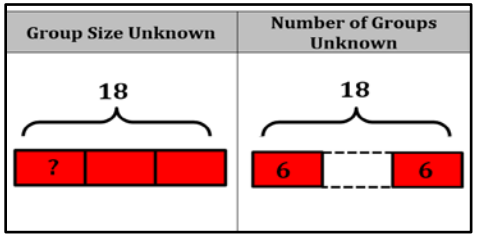
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.3** Use multiplication and division to solve word problems involving equal groups, arrays, and measurement quantities **3.OA.4** Determine the unknown number in a multiplication or division equation relating three whole numbers **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand both division as an unknown-factor problem and the relationship between multiplication and division **3.OA.7** Fluently multiply and divide	**4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison **4.OA.3** Solve multi step word problems with all operations **4.NBT.6** Find whole-number quotients with up to 4-digit dividends and 1-digit divisors **5.NBT.6** Find whole-number quotients with up to 4-digit dividends and 2-digit divisors **5.NBT.7** Solve equations involving decimals with all operations **5.NF.7** Apply and extend previous understandings of division to fractions

Critical Background Knowledge from Previous Grade Levels

- Add and subtract within 20 (2.OA.2)
- Use addition to find the total number of objects arranged in an array (2.OA.4)

Academic Vocabulary

quotient, dividend, divisor, divide, equal groups, whole numbers

Suggested Models	Suggested Strategies
Partitive Division: There are 12 cookies. If you put them in three bags, how many cookies will be in each bag?  Quotative Division: There are 12 cookies. If you give put 3 cookies in each bag, how many bags will you fill?   Partitive Quotative	● Use manipulatives/objects or other models ● Use repeated subtraction ● Drawing pictures ● Model equal groups ● Model equal groups with various counters ● Discuss real-life situations where objects are in groups ● Use and compare number lines, bar models, and area models

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *For example, use drawings and equations with a symbol for the unknown number to represent the problem.*

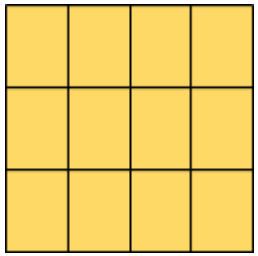
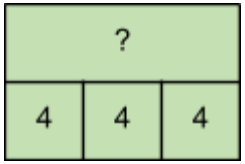
Concepts and Skills to Master

- Determine the operation based on the situation in the context of a word problem (avoid relying on keyword strategies)
- Use numbers and symbols to represent word problems (×, ÷, =, and a variety of symbols for unknowns)
- Solve the following multiplication and division situations. (See: TABLE 2. Common multiplication and division situations):
  - **Equal Groups of Objects/Product Unknown** word problems (There are 3 bags with 4 plums in each bag. How many plums are there in all?)
  - **Equal Groups of Objects/Group Size Unknown** word problems (24 plums are shared equally into 3 bags. How many plums will be in each bag?)
  - **Equal Groups of Objects/Number of Groups Unknown** word problems (24 plums are packed equally into some bags. 8 plums are packed into each bag. How many bags are needed?)
  - **Arrays of Objects/Product Unknown** word problems (The apples in the grocery window are in 3 rows and 4 columns. How many apples are there?)
  - **Arrays of Objects/Group Size Unknown** word problems (If 12 apples are arranged into an array with 3 rows, how many columns of apples are there?)
  - **Arrays of Objects/Number of Groups Unknown** word problems (If 12 apples are arranged into an array with 4 columns, how many rows are there?)

Teacher Note: In this standard emphasis should be placed in solving for products of two one-digit numbers. Students may also be expected to solve problems in which a two-digit number is multiplied by a one-digit with a product less than or equal to 100. Emphasis should be placed on one-digit numbers multiplied by one-digit numbers; however, students should be exposed to a variety of problems with products less than or equal to 100. Examples may include problems such as: 12 × 5 = 60, 25 × 4 = 100, 33 × 3 = 99, etc. Multiplicative comparison situations (35 is 5 times as many as 7 and 7 times as many as 5) should not be introduced in third grade. This concept will be introduced in fourth grade in Standards 4.OA.1 and 4.OA.2.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1, **3.OA.2** Interpret products of whole numbers and whole-number quotients **3.OA.4** Determine the unknown whole number in a multiplication or division equation **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand the relationship between multiplication and division **3.OA.7** Fluently multiply and divide within 100 **3.OA.8** Solve two-step word problems **3.MD.2** Multiply and divide to solve measurement word problems **3.MD.7** Relate area to multiplication	**4.OA.2** Multiply and divide to solve word problems involving multiplicative comparisons **4.OA.3** Solve multi-step word problems using whole numbers and having whole-number answers using the four operations **4.NBT.5, 4.NBT.6** Multiply and divide with multi-digit numbers **4.NF.4** Extend understandings of multiplication to multiply a fraction by a whole number **5.NF.4, 5.NF.6, 5.NF.7** Extend understandings of multiplication and division to multiply and divide with fractions **5.NBT.5** Fluently multiply multi-digit whole numbers **5.NBT.6** Find whole-number quotients

Critical Background Knowledge from Previous Grade Levels
• Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4) • Partition a rectangle into rows and column of same-sized squares and count to find the total number of squares (2.G.2) • Use addition and subtraction to solve word problems (1.OA.1, 2.OA.1)
Academic Vocabulary
equal groups, array, row, column, area model, multiply, product, factor, divide, quotient, divisor, dividend

Suggested Models	Suggested Strategies
$3 \times 4 = 12$ 3 groups of 4 is 12  4 + 4 + 4 + 4 = 12 $4 \times 3 = 12$  4 $4 \times 3 = 12$ 3  $4 \times 3 =$? $4 \times 3 = 12$ 	• Use objects and drawings to represent equal groups and arrays; Describe factors, products, etc. in these models • Use bar models • Use counting all, skip counting, repeated addition to multiply • Write equations to represent drawings and objects; Explain connections between physical/visual models and equations • Use the relationship between multiplication and division to solve problems • Use a multiplication strategy (compensation, distributive property) to solve word problems • Apply the commutative or associative properties of multiplication • Students may create their own word problems • Use equal groups, arrays, area models, bar models to solve problems • Use repeated subtraction to divide

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number— product, factor, quotient, dividend, or divisor—that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?.*

Concepts and Skills to Master

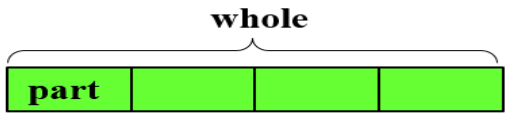
- Solve the following multiplication and division situations (See: TABLE 2. Common multiplication and division situations):
  - **Equal groups / unknown product** word problems (There are 3 bags with 6 plums in each bag. How many plums are there in all?)
  - **Equal groups / group size unknown** word problems (If 18 plums are shared equally into 3 bags, then how many plums will be in each bag)
  - **Equal groups / number of groups unknown** word problems (If 18 plums are to be packed 6 to a bag, then how many bags are needed?)
  - **Array or area / unknown product** word problems (There are 3 rows of apples with 6 apples in each row. How many apples are there?)
  - **Array or area / group size unknown** word problems (If 18 apples are arranged into 3 equal rows, how many apples will be in each row?)
  - **Array or area / number of groups unknown** word problems (If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?)
- Understand that equations involving multiplication and division relate three whole numbers in related facts (3 × __ = 15; 15 ÷ __ = 3; 15 ÷ 3 = __)
- Use a symbol to represent an unknown number
- Apply multiplication or division to solve for an unknown in an equation

Teacher Note: Comparison problem types are not introduced until 4th grade. Equations in the form of $a \times b = c$ and $c = a \times b$ should be used interchangeably, with the unknown in different positions. Examples: 24 = ? × 6, 72 ÷ __ = 9, or the following problem: Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 × 4 = m

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.3 Use multiplication and division within 100 with symbols for the unknown number **3.OA.7** Fluently multiply and divide using the relationship between multiplication and division **3.MD.8** Solve real-world and mathematical problems involving perimeters	**4.NBT.5** Multiply a whole number of up to four digits **4.NBT.6** Find whole number quotients **4.OA.3** Solve multi-step word problems posed with whole numbers **4.OA.2** Multiply or divide to solve word problems **4.MD.3** Apply the area and perimeter formulas for rectangles; view the area formula as a multiplication equation with an unknown factor **5.NBT.5** Fluently multiply multi-digit whole numbers **5.NBT.6** Find whole digit quotients using the relationship between multiplication and division

Critical Background Knowledge from Previous Grade Levels

- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4)
- Partition a rectangle into rows and columns of same-sized squares and count to find the total number of squares (2.G.2)
- Use addition and subtraction within 20 to solve word problems involving situations with unknowns in all positions. (1.OA.1)
- Understand the meaning of the equal sign (1.OA.7)
- Determine the unknown whole number in an addition and subtraction equation relating three whole numbers (1.OA.8)

Academic Vocabulary	
symbol, equal, multiplication, product, factor, quotient, dividend, divisor, division	
Suggested Models	**Suggested Strategies**
Part Part Whole/Multiplication and Division  one part × number of parts = whole whole ÷ number of parts = one part	● Use a bar model to solve for the unknown whole number in an equation ● Use counters to model the relationship between multiplication and division ● Use base ten blocks to represent array and area models ● When given an equation such as 4 × ? = 40, students explain their thinking, for example: - 4 groups of some number is the same as 40 - 4 times some number is the same as 40 - I know 4 groups of 10 is 40 so the unknown number is 10 - • The missing factor is 10 because 4 times 10 equals 40

Demonstrate understanding of the properties of multiplication and the relationship between multiplication and division (Standards 3.OA.5–6).

Standard 3.OA.5 Apply properties of operations as strategies to multiply and divide. *For example: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (commutative property of multiplication). 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (associative property of multiplication). Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (distributive property).* (Third grade students may, but need not, use formal terms for these properties.)

Concepts and Skills to Master

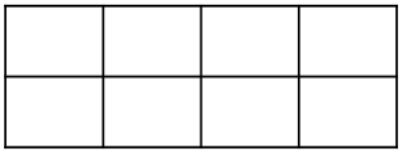
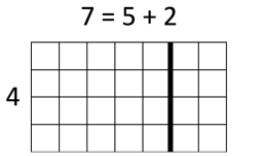
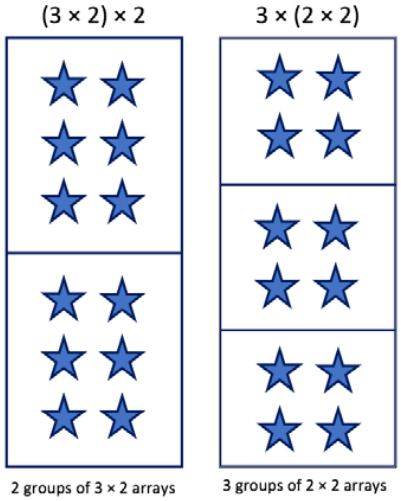
- Understand that multiplication is commutative and division is not commutative (the order of the factors does not change the product of an equation)
- Understand and apply the associative property of multiplication (factors can be grouped differently without changing the product)
- Understand and apply the distributive property of multiplication over addition (to support students in solving for products by breaking apart the numbers)
- Understand and apply the multiplicative identity property of one (8 × 1 = 8)
- Understand and apply the zero property of multiplication (8 × 0 = 0)
- Apply properties to simplify an expression into smaller problems (3 × 7 = (3 × 2) + (3 × 5); 3 × 8 = 3 × 2 × 4)

Teacher Note: Emphasis should be placed on understanding of the properties and why each property applies to a particular operation rather than memorizing names and definitions. Convention defines arrays as rows by columns, however students should be allowed flexibility in describing arrays as either rows by columns or columns by rows and should understand how rotating an array demonstrates the commutative property.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.2** Interpret whole-number quotients of whole numbers **3.OA.3** Use multiplication and division to solve word problems **3.OA.4** Determine the unknown whole number in a multiplication or division equation **3.OA.6** Understand division as an unknown-factor problem **3.OA.7** Fluently multiply and divide **3.MD.7** Relate area to the operations of multiplication and addition **3.OA.8** Solve two-step word problem **3.OA.9** Identify arithmetic patterns and explain them using properties of operations **3.NBT.3** Multiply one-digit whole numbers by multiples of 10 using strategies based on place value and properties of operations	**4.NBT.5** Multiply whole numbers using strategies based on the properties of operations **4.NBT.6** Find whole-number quotients and remainders based on the properties of operations **4.OA.3** Solve multi-step word problems **5.OA.1** Use parenthesis, brackets, and braces in numerical expressions **5.MD.5** Relate volume to the operations of multiplication and addition

Critical Background Knowledge from Previous Grade Levels

- Explain why addition and subtraction strategies work, using place value and the properties of operations (2.NBT.9)
- Use addition to find the total number of objects in a rectangular array (2.OA.4)
- Apply properties of operations as strategies to add and subtract (2.NBT.5, 1.OA.3, 1.NBT.4)

Academic Vocabulary	
multiplication, division, product, factor, dividend, divisor, quotient, commutative property of multiplication, associative property of multiplication, parentheses, distributive property of multiplication over addition, zero property of multiplication, multiplicative identity property of one, array	
Suggested Models	Suggested Strategies
Commutative Property of Multiplication Model  This array can be seen as rows by columns (2 x 4) or columns by rows (4 x 2). **Distributive Property of Multiplication Model** $4 \times 7 = 4 \times (5 + 2)$ $= (4 \times 5) + (4 \times 2)$ $= 20 + 8$ $= 28$  7 = 5 + 2 4 (4 × 5) + (4 × 2) = 4 × 7 20 + 8 = 28 **Associative Property of Multiplication Model**  (3 × 2) × 2 3 × (2 × 2) 2 groups of 3 × 2 arrays 3 groups of 2 × 2 arrays	• Use an array or grouping to model the commutative property • Model the distributive property of multiplication over addition (see Suggested Models) • Use base-ten blocks, multiplication charts, grid/graph paper, and area models

Demonstrate understanding of the properties of multiplication and the relationship between multiplication and division (Standards 3.OA.5–6).

Standard 3.OA.6 Understand division as an unknown-factor problem. Understand the relationship between multiplication and division (multiplication and division are inverse operations). *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Concepts and Skills to Master

- Understand the relationship between multiplication and division as inverse operations, one operation can help solve the other
- Understand and solve unknown-factor problems
- Solve a division equation by using related multiplication facts

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.2** Interpret whole-number quotients of whole numbers **3.OA.3** Use multiplication and division within 100 to solve word problems **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.7** Fluently multiply and divide **3.OA.8** Solve two-step word problem **3.OA.9** Identify arithmetic patterns and explain them using properties of operations **3.MD.7** Relate area to the operations of multiplication and addition	**4.NBT.6** Find whole-number quotients and remainders based on place value, the properties of operations, and the relationship between multiplication and division **5.NF.3** Interpret a fraction as division, solving real-world problems involving division of whole numbers **5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers **5.NF.7** Apply and extend previous understandings of division to unit fractions and whole numbers

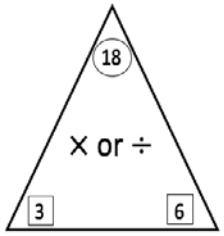
Critical Background Knowledge from Previous Grade Levels

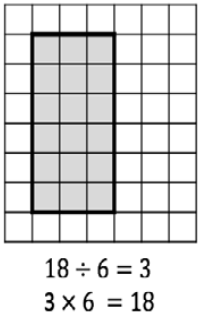
- Use addition and subtraction with unknowns in all positions (2.OA.1)
- Use addition and subtraction within 20 to solve word problems involving situations with unknowns in all positions. (1.OA.1)
- Determine the unknown whole number in an addition and subtraction equation relating three whole numbers (1.OA.8)

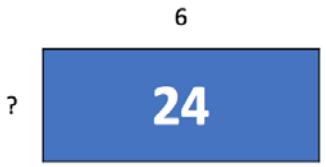
Academic Vocabulary

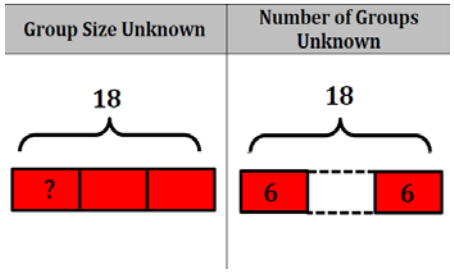
related facts, multiplication, division, inverse operation, factor

Suggested Models









Suggested Strategies

- Use fact families and/or number bonds
- Model arrays to show related multiplication and division equations (e.g., 3 × 2 = 6; 2 × 3 = 6; 6 ÷ 2 = 3; 6 ÷ 3 = 2).
- Use equal groups, number lines, and area models

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.7 Fluently multiply and divide.

a. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. *(For example, knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8)*.

b. By the end of Grade 3, know from memory all products of two one-digit numbers.

Concepts and Skills to Master

- Apply multiplication and division strategies flexibly, accurately and efficiently
- Understand the inverse relationship of multiplication and division
- Understand and apply commutative and distributive properties
- Know from memory all products of two one-digit numbers

Teacher Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms. Students develop fluency over time as they have repeated experiences that build conceptual understanding of multiplication (concrete and pictorial representations, patterns, context, etc.). Learning is enhanced when practice is organized to focus most heavily on understood but not yet fluent facts. Fluency may be reached by becoming fluent for each number (2s, 5s, etc. by noticing patterns, not through memorization) and then extending the fluency to several, then all numbers mixed together. To achieve fluency by the end of third grade, students must begin working toward fluency as early as possible. This is not a matter of instilling facts divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning. (Adapted from: https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf, p. 27)

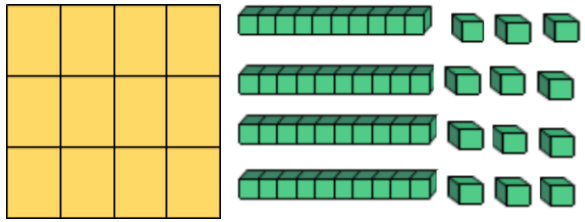
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.2** Interpret whole-number quotients **3.OA.3** Use multiplication and division within 100 to solve word problems **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand division as an unknown-factor problem	**4.OA.4** Find all factor pairs for a whole number between 1-100 **4.NBT.5** Multiply up to four-digit numbers by one-digit numbers and two-digit numbers by two-digit numbers **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors **5.NBT.5** Fluently multiply multi-digit whole numbers **5.NBT.6** Find whole-number quotients **4.OA.1–3, 4.NF.1–2 and 4, 5.NF.4 and 6–7** Fluency with multiplication is a foundation for extending strategies when multiplying and dividing multi-digit whole numbers, fractions, and decimals

Critical Background Knowledge from Previous Grade Levels

- See Related Standards: Current Grade Level
- Fluently add and subtract within 20 (2.OA.2)
- Work with equal groups (2.OA.4)
- Partition rectangles into squares (2.G.2)
- Apply properties of operations as strategies to add and subtract (1.OA.3)

Academic Vocabulary

product, factor, dividend, divisor, quotient, multiplication, multiply, division, divide, commutative property of multiplication, distributive property

Suggested Models	Suggested Strategies
See models listed in the Core Guide for 3.OA.3 as students work to build fluency. Area model for 3 × 4 Base ten blocks used to represent 4 × 13  **Patterns in multiples of 9** $1 \times 9 = 9$ $2 \times 9 = 2 \times (10 - 1) = (2 \times 10) - (2 \times 1) = 20 - 2 = 18$ $3 \times 9 = 3 \times (10 - 1) = (3 \times 10) - (3 \times 1) = 30 - 3 = 27,$ etc	• Model and/or count • Apply the Commutative Property • Find missing factors • Engage in number talk or math discourse • Play games for practice • Analyze multiplication by zeros and ones • Skip count (counting groups of __ and knowing how many groups have been counted) • Use doubles (2s), doubling twice (4s), doubling three times (8s) • Use tens facts (relating to place value, 5 × 10 is 5 tens or 50) • Use five facts (half of tens) • Recognize square numbers (e.g., 3 × 3) • Identify patterns in multiples of nines (10 groups less one group, e.g., 9 × 3 is 10 groups of 3 minus one group of 3) • Decompose into known facts (6 × 7 is 6 × 6 plus one more group of 6) • Use related facts (e.g., 6 × 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 × 6 = 24) • Recognize and use patterns in multiplication table

Fluently multiply and divide within 100

Knowing from memory all products of two one-digit numbers includes the following facts:

x	0	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Fluency involves a mixture of just knowing some answers, knowing some answers from patterns (for example, multiplying by one yields the same number), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students.

Emphasis should be placed on one-digit numbers multiplied by one-digit numbers; however, students should be exposed to a variety of problems with products less than or equal to 100. Students are expected to use concrete models and reasoning strategies to solve problems in which a two-digit number is multiplied by a one-digit with a product less than or equal to 100. Examples may include problems such as: 15 × 5 = 75, 25 × 4 = 100, 33 × 3 = 99, etc. The standard algorithm for multiplication is introduced in fifth grade in standard 5.NBT.5 and should not be taught in third grade.

Text Source: https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

Students use the four operations to identify and explain patterns in arithmetic (Standards 3.OA.8–9).

Standard 3.OA.8 Solve two-step word problems.

a. Solve two-step word problems using the four operations. Know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (Limit to problems posed with whole numbers and having whole number answers.)

b. Represent two-step problems using equations with a letter standing for the unknown quantity. Create accurate equations to match word problems.

c. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

Concepts and Skills to Master

- Differentiate between one-step and two-step word problems (Two-step word problems may include any combination of two operations in the same problem)
- Determine the operation(s) based on the actions in the context of two-step word problems (avoid relying on keyword strategies)
- Use numbers and symbols to represent word problems (+, -, ×, ÷, =, and a letter for unknowns)
- Know that multiplication and division are performed (in the order they appear in the problem; from left to right) prior to addition and subtraction (in the order they appear in the problem; from left to right)
- Solve and apply the addition, subtraction, multiplication, and division situations listed in Standards K.OA.2, 1.OA.1, and 2.OA.1, and 3.OA.3

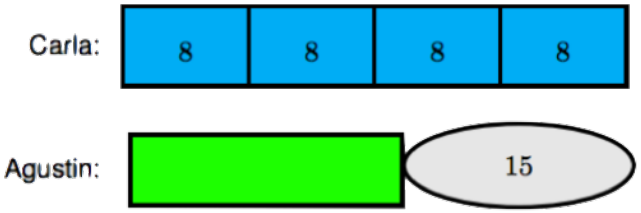
Related Standards: Current Grade Level	Related Standards: Future Grade Level
3.OA.1, 3.OA.2 Interpret products of whole numbers and whole-number quotients **3.OA.4** Determine the unknown whole number in a multiplication or division equations **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand the relationship between multiplication and division **3.OA.7** Fluently multiply and divide **3.OA.8** Solve two-step word problems **3.MD.2** Multiply and divide to solve measurement word problems **3.MD.7** Relate area to multiplication	**4.OA.2** Multiply and divide to solve word problems involving multiplicative comparisons **4.OA.3** Solve multi-step word problems using whole numbers and having whole-number answers using the four operations **5.NF.4** Apply and extend previous understandings of multiplication and division to multiply and fraction or a whole number by a fraction

Critical Background Knowledge from Previous Grade Levels

- Interpret products of whole numbers and whole-number quotients (3.OA.1, 3.OA.2)
- Understand and use the associative and commutative properties
- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4)
- Partition a rectangle into rows and column of same-sized squares and count to find the total number of squares (2.G.2)
- Use addition and subtraction to solve word problems (1.OA.1, 2.OA.1)

Academic Vocabulary

Addends, sum, difference, round, estimate, equation, difference, multiplication, factors, product, array, multiples, division, divisor, dividend, quotient, reasonableness, symbol, ×, ÷, /

Suggested Models	Suggested Strategies
A two-step problem with diagram showing problem situation and equations showing the two parts Carla has 4 packages of silly bands. Each package has 8 silly bands in it. Agustin is supposed to get 15 fewer silly bands than Carla. How many silly bands should Agustin get?  C = number of Carla's silly bands A = number of Agustin's silly bands $C = 4 \times 8 = 32$ $A + 15 = C$ $A + 15 = 32$ $A = 17$ *Students may be able to solve this problem without writing such equations.*	• Use drawings, objects, and equations • Use a bar model • Apply Part/Part/Whole • Create student-generated word problems • Skip count • Use the relationship between multiplication and division
Image Source: https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf	

Students use the four operations to identify and explain patterns in arithmetic (Standards 3.OA.8–9).

Standard 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that four times a number is always even, and explain why four times a number can be decomposed into two equal addends.*

Concepts and Skills to Master

- Recognize arithmetic patterns that can be found on a hundreds chart, a number line, an addition and a multiplication table
- Recognize multiplication patterns that can be found on a hundreds chart and a multiplication table
- Know that multiplication by an even number results in an even number
- Know that multiplication of an odd number by another odd number results in an odd number
- Know that multiplication of an odd number by an even number results in an even number
- Explain arithmetic patterns using properties of operations
- Find the products of the commutative property on the multiplication chart
- Model addition and multiplication patterns with a number line, hundreds chart, multiplication chart

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.5 Apply properties of operations as strategies to multiply and divide	**4.OA.5** Generate number or shape patterns that follow a given rule **5.OA.3** Generate two numerical patterns using two given rules

Critical Background Knowledge from Previous Grade Levels

- Determine whether a group of objects is odd or even (2.OA.3)
- Recognize patterns of skip counting with fives, tens, and hundreds (2.NBT.2)

Academic Vocabulary

sum, multiplication, multiples, factors, product, sequence, pattern , row, column

Suggested Models	Suggested Strategies
Highlight a given factor and discuss patterns noticed	● Use number lines ● Use hundreds charts ● Highlight and discuss patterns on multiplication and addition charts ● Analyze patterns in basic facts

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Patterns in multiples of 9

$1 \times 9 = 9$

$2 \times 9 = 2 \times (10 - 1) = (2 \times 10) - (2 \times 1) = 20 - 2 = 18$

$3 \times 9 = 3 \times (10 - 1) = (3 \times 10) - (3 \times 1) = 30 - 3 = 27,$ etc

TABLE 2. Common multiplication and division situations. [1]

	Unknown Product **3 × 6 = ?**	Group Size Unknown *("How many in each group?" Division)* **3 × ? = 18 and 18 ÷ 3 = ?**	Number of Groups Unknown *("How many groups?" Division)* **? × 6 = 18 and 18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example.*** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example.*** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example.*** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2]	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
AREA[3]	What is the area of a 3 cm by 6 cm rectangle?	A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE[4]	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? ***Measurement example.*** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? ***Measurement example.*** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? ***Measurement example.*** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[4] Multiplicative Compare problems appear first in Grade 4, with whole-number values in all places, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs A times as much as the blue hat" results in the same comparison as "A blue hat costs 1/A times as much as the red hat," but has a different subject.

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of algorithms may be used (Standards 3.NBT.1–3).

Standard 3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

Concepts and Skills to Master

- Use place value understanding to round two-digit and three-digit numbers to the nearest 10
- Use place value understanding to round two-digit and three-digit numbers to the nearest 100
- Understand when rounding to the nearest 10 or 100, the goal is to approximate the closest number with zero ones or zero tens and ones (For example, 478 rounded to the nearest ten is 480; and 478 rounded to the nearest hundred is 500)
- Connect rounding numbers to the location of the number on a number line by identifying the benchmark numbers and using the midpoint to determine which benchmark number is closer (For example, when rounding 478 to the nearest ten, the benchmark numbers are 470 and 480. The midpoint is 475. The number 478 is to the right of the midpoint and closer to 480 than 470. The number 478 is therefore rounded to 480.)

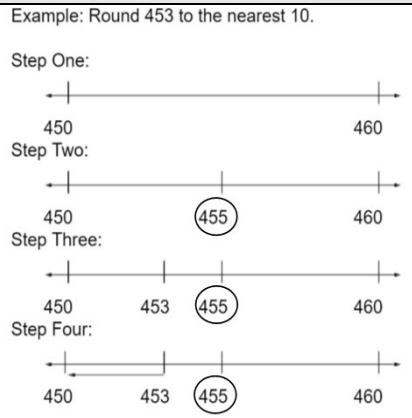
Teacher Note: Third grade is the first time students round numbers. Rounding to the unit represented by the place farthest to the left is typically easier for students and is often sufficient for practical purposes. Rounding to the unit represented by a place in the middle of a number may be more difficult for students as the surrounding digits are sometimes distracting. For example, it may be easier for a student to round 478 to 500 rather than to 480. Students should have experience rounding three-digit numbers to both the nearest 10 and nearest 100.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.8 Solve two-step word problems and assess the reasonableness of answers using mental computation and estimation strategies including rounding	**4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place up to 1,000,000 **4.OA.3** Solve multi-step word problems and assess the reasonableness of answers using mental computation and estimation strategies including rounding **5.NBT.4** Use place value understanding to round decimals to hundredths

Critical Background Knowledge from Previous Grade Levels

- Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form (2.NBT.3)
- Understand that the three-digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand the value of each digit in three-digit numbers (2.NBT.1)
- Understand that the two-digits of a two-digit number represent amounts of tens, and ones. Understand the value of each digit in two-digit numbers (1.NBT.2)

Suggested Models



Academic Vocabulary

round, benchmark number, midpoint, digits, estimate, close to, nearest ten, tens place, nearest hundred, hundreds place

Suggested Strategies

- Create and use horizontal and vertical open number lines to identify, locate, and label benchmark numbers, midpoints, and target numbers to assist in rounding
- Use base-ten blocks and drawings to model the concept of rounding
- Use a hundreds chart or place value chart as tools for support when rounding
- While songs and mnemonic stories may be engaging, they should not be used in place of developing conceptual understanding of rounding; If these are to be used, they should come after conceptual understanding has been developed

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of algorithms may be used (Standards 3.NBT.1–3).

Standard 3.NBT.2 Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Concepts and Skills to Master

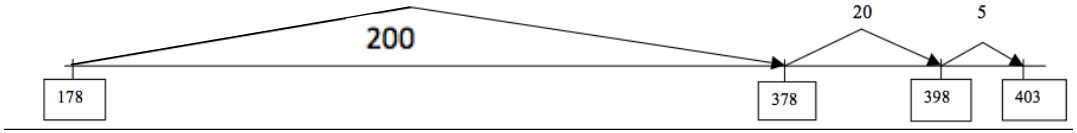
- Add and subtract fluently within 1,000 using strategies based on place value
- Use multiple strategies and algorithms fluently to add and subtract within 1,000
- Explain why addition and subtraction strategies work when adding and subtracting within 1,000
- Identify when it is necessary to compose (regroup) or decompose (ungroup) a ten or hundred
- Decompose a ten to subtract a two-digit number from a two- or three-digit number
- Decompose a hundred to subtract a three-digit number from a three-digit number
- Write equations for addition and subtraction with sums and differences within 1,000
- Understand how to compute sums and difference in a variety of situations, including with zeros in various places
- Understand and use the commutative property and associative property when adding and subtracting
- Understand the inverse relationship between addition and subtraction to fluently add and subtract within 1,000

Teacher Note: This standard builds on students work with 2.NBT.7, where students operate with values within 1,000. In third grade, students should become more fluent in these operations. The standard algorithm of compose and decompose is neither an expectation nor a focus in third grade. Students use multiple strategies for addition and subtraction in grades K-3. By the end of third grade students use a range of algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction to fluently add and subtract within 1000. Students are expected to fluently add and subtract multi-digit whole numbers using the standard algorithm by the end of fourth grade.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.8 Solve two-step word problems **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table) explain them using properties of operations.	**4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted **4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm

Critical Background Knowledge from Previous Grade Levels

- Fluently add and subtract within 20 (2.OA.2)
- Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (2.NBT.5)
- Add and subtract within 1,000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, and ones and ones, and that it is sometimes necessary to compose or decompose tens or hundreds (2.NBT.7)
- Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900 (2.NBT.8)
- Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects (2.NBT.9)
- Apply properties of operations as strategies to add and subtract (1.OA.3)

Academic Vocabulary	
compare, digits, expanded form, place value, standard form, word form, addends, sum, commutative (order) property of addition, identity (zero) property of addition, associative property of addition (grouping), fact family, difference, equation	
Suggested Models	**Suggested Strategies**
Example: There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground? Student 1: 100 + 200 = 300 70 + 20 = 90 8 + 5 = 13 300 + 90 + 13 = 403 students Student 2: I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403. Student 3: I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403. Student 4: 178 + 225 = ? 178 + 200 = 378 378 + 20 = 398 398 + 5 = 403  	• Use hundreds chart to add and subtract • Use base ten blocks to add and subtract • Use an open number line to add and subtract • Use physical models to add and subtract • Use place value charts to add and subtract • Use mental computation to develop conceptual understanding and number sense adding and subtracting two and three digit numbers
Images Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/2.pdf	

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of algorithms may be used (Standards 3.NBT.1–3).

Standard 3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 *(for example, 9 x 80 and 5 x 60)* using strategies based on place value and properties of operations.

Concepts and Skills to Master

- Recognize when a number is a multiple of 10
- Represent the product of a one-digit number multiplied by a multiple of ten as groups of tens (Represent 3 × 50 as 3 groups of 5 tens, which is 15 tens or 150)
- Use the associative and/or distributive property of multiplication to explain the patterns when multiplying by multiples of ten (3 × 50 = 3 × (5 × 10) = (3 × 5) × 10 = 15 × 10 = 150)
- Generalize what happens when a one-digit number is multiplied by a multiple of ten (the non-zero digits appear to shift to the left with a zero in the ones place)

Teacher Note: This is an introductory year for multiplication. Third grade students work with understanding multiplication in the OA standards. This standard supports relating multiplication and place value. In fourth grade, students work with multi-digit multiplication using strategies based on place value (4.NBT.5).

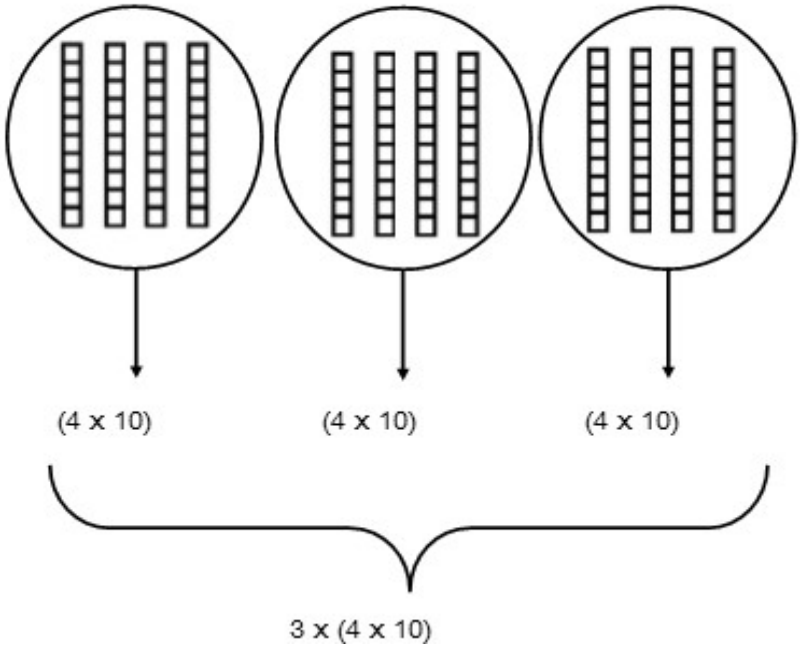
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers, such as interpreting 5 × 7 as the total number of objects in 5 groups of 7 objects each **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. By the end of third grade, know from memory all products of two one-digit numbers	**4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right **4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers **5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm

Critical Background Knowledge from Previous Grade Levels

- Related Standards: Current Grade Level (see above)
- Skip-count by tens within 1,000 (2.NBT.2)
- Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900 (2.NBT.8)
- Given a two-digit number, mentally find 10 more than the number (1.NBT.5)

Academic Vocabulary

multiplication, factor, product, equal groups, array, multiple of 10, place value

Suggested Models	Suggested Strategies
Example: Model 3 x 40 with base-ten blocks.  	• Use a variety of strategies to represent multiplication (base-ten blocks, drawings, equal groups, arrays, area models, number lines, and/or hundreds charts) • Extend strategies for one-digit factors to multiply with groups of tens • Use open number lines (4 x 20 - make jumps at 20, 40, 60, 80) • Discuss patterns and make generalizations

Develop understanding of fractions as numbers. Denominators are limited to 2, 3, 4, 6, and 8 in third grade.

Standard 3.NF.1 Understand that a unit fraction has a numerator of one and a non-zero denominator.

a. Understand a fraction 1/*b* as the quantity formed by one part when a whole is partitioned into *b* equal parts.

b. Understand a fraction *a/b* as the quantity formed by a parts of size 1/*b*. *For example: 1/4 + 1/4 + 1/4 = 3/4.*

Concepts and Skills to Master

- Understand unit sized fractional parts as equal-sized pieces of the same whole
- Understand a unit fraction as one of the equal-sized parts of the whole with a one as the numerator
- Understand the denominator as the fractional name determined by the number of equal parts in the whole
- Reason about the size of the fractional part in relation to the number of parts in a whole
- Understand the numerator of a fraction as the number of equal parts being considered
- Build non-unit fractions from unit fractions (¾ is composed of 3 units the size of ¼)

Teacher Note: Third grade is the first time students work with fractions as numbers, including fractional notation as well as representations beyond circles and rectangles. Students should NOT be exposed to representations of fractions as sets until fourth grade.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.NF.2 Understand fractions on number lines **3.NF.3** Explain equivalence and compare fractions **3.G.2** Partition shapes into parts with equal areas	**4.NF.3** Understand fractions as sums of unit fractions **4.NF.1, 2, 4, 5.NF.1–6** Use understanding of fractions and unit fractions to solve operations with fractions **5.NF.7** Divide unit fractions by whole numbers and whole numbers by unit fractions

Critical Student Background Knowledge from Previous Grade Levels

- Partition circles and rectangles into equal shares using the language halves, thirds, and fourths (2.G.3)
- Partition circles and rectangles into equal shares using the language halves and fourths (1.G.3)

Academic Vocabulary

halves (1/2), thirds (1/3), fourths (1⁄4), sixths (1/6), eighths (1⁄8), fraction, unit fraction, numerator, denominator, equal parts

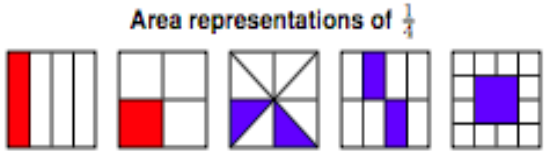
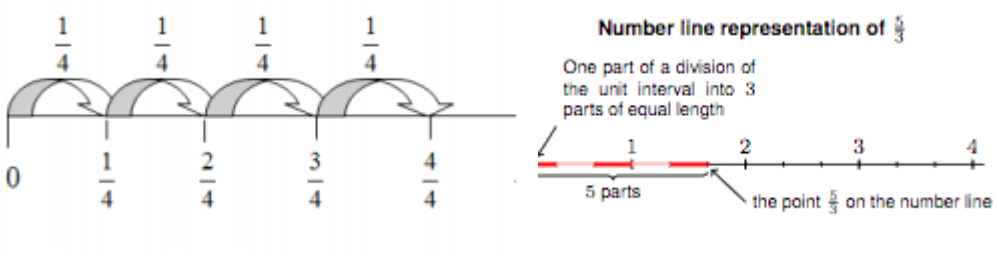
Suggested Models	Suggested Strategies
 *In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.*	• Represent fractions using various contexts (candy bars, fruits, cake), materials (paper, objects), and shapes (circles, squares, rectangles, strips, fraction bars) • Represent fractions using area of shapes, number lines, and sets of objects • Represent unit fractions and non-unit fractions connecting visual models to fractional notation

Image Source: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf

Develop understanding of fractions as numbers. Denominators are limited to 2, 3, 4, 6, and 8 in third grade.
Standard 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. **a.** Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. **b.** Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.
Concepts and Skills to Master
• Understand that the interval from 0 to 1 or the interval between consecutive whole numbers can represent a whole • Recognize the equal parts as unit fractions when the whole between 0 and 1, on a number line has been partitioned into equal parts • Understand the endpoint labels the length and the fraction • Identify and represent fractions on a number line

Related Standards: Current Grade Level	
3.NF.1 Understand unit fractions and fractions as numbers **3.NF.3** Explain equivalence and compare fractions **3.G.2** Partition shapes into parts with equal areas **3.MD.1** Represent time intervals on a number line **3.MD.4** Measure lengths with halves and fourths of an inch	**4.NF.3** Understand fractions as sums of unit fractions **4.NF.1, 2, 4, 5.NF.1 & 2, 5.NF.4 - 7** Use number lines as models to represent operations with fractions **4.MD.4, 5.MD.2** Make line plots with fractional measurements **5.G.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system **6.NS.5,6,7** Use number lines to reason about and compare positive and negative numbers

Critical Background Knowledge from Previous Grade Levels
Represent whole numbers as lengths from 0 on a number line diagram (2.MD.6)
Academic Vocabulary
halves (1/2), thirds (1/3), fourths (1/4), sixths (1/6), eighths (1/8), fraction, unit fraction, numerator, denominator, equal parts, interval, endpoint

Suggested Models	Suggested Strategies
	• Use a variety of linear models including folding paper strips (sentence strips), string, etc. to reason and justify the location of fractions on a number line • Connect physical models, to visual bar models, and number lines • Make connections between number lines, rulers, and time intervals (3.MD.1 and 3.MD.4)

Images Sources: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf, http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/3.pdf

Develop understanding of fractions as numbers. Denominators are limited to 2, 3, 4, 6, and 8 in third grade.

Standard 3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, such as 1/2 = 2/4, 4/6 = 2/3. *Explain why the fractions are equivalent by using a visual fraction model, for example*.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. *Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, for example, by using a visual fraction model.*

Concepts and Skills to Master

- Understand equivalent fractions as the same quantity with different names
- Understand equivalence as different names for the same point on a number line
- Represent whole numbers as equivalent fractions (3/3 = 1 and 4/1 = 4)
- Understand comparisons are only valid when the two fractions refer to the same whole
- Compare unit fractions by reasoning that as the number of equal parts in a whole increases, the size of the fractional parts decreases (the larger the denominator, the smaller the size of the part, ex. ½ > ⅛)
- Compare non-unit fractions with the same numerators by reasoning that as the number of equal parts in a whole increases, the size of the fractional parts decreases. The larger the denominator, the smaller the size of the part. (2/4 > 2/6)
- Compare fractions with the same denominators by reasoning that as the number of equal parts being considered (numerator) increases, the size of the fraction increases. The greater numerator is greater because it is made of more unit fractions. (A segment from 0 to ¾ is shorter than a segment from 0 to 5/4, because it measures 3 units of ¼ as opposed to 5 units of ¼. Therefore, ¾ < 5/4.)

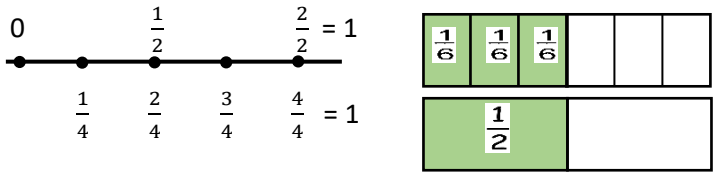
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.NF.1 Understand unit fractions and fractions as numbers **3.NF.2** Understand fractions on number lines	**4.NF.1** Generate equivalent fractions, and explain why they are equivalent **4 NF.2** Compare and order fractions by generating equivalent fractions **5.NF.1, 5.NF.2** Add and subtract fractions with unlike denominators, by generating equivalent fractions **6.RP.3** Generate equivalent ratios and compare ratios

Critical Background Knowledge from Previous Grade Levels

- Compare two-digit and three-digit numbers with the symbols >, =, and < (1.NBT.3, 2.NBT.4)
- Measure an object using different units and relate the number of units to the size of the units. The larger the size of the unit, the less units needed. A book is 1 foot or 12 inches. A foot is larger so less feet are needed. Inches are smaller so more inches are needed (2.MD.2)
- Understand that decomposing into more equal shares creates smaller shares (1.G.3)
- Order and compare objects by length (1.MD.1)

Academic Vocabulary

halves (1/2), thirds (1/3), fourths (1/4), sixths (1/6), eighths (1/8), fraction, numerator, denominator, equivalent, equal parts, compare

Suggested Models	Suggested Strategies
Using the number line and fraction strips to see fraction equivalence  Visual models may include: area of various shapes (circles, rectangles, etc.), bar models, number lines, and double number lines	• Use a variety of visual area and linear fraction models to recognize and generate equivalent fractions. • Use a variety of visual area and linear fraction models to compare fractions with the same numerators and same denominators. • Use objects of different sizes and discuss if the fractions may be compared. Is ½ of a small Laffy Taffy the same amount as ½ of a large Laffy Taffy?

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (Standards 1–2).

Standard 3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. *Solve word problems involving addition and subtraction of time intervals in minutes, for example, by representing the problem on a number line diagram.*

Concepts and Skills to Master

- Understand there are 60 minutes in an hour and view an hour in intervals of one, five, fifteen, and thirty minutes
- Represent and write time to the nearest minute on analog and digital clocks using a.m. and p.m.
- Understand the relationship between a clock and a number line and represent problems involving time on a number line diagram
- Measure time intervals (elapsed time) in minutes
- Solve word problems involving addition and subtraction of time intervals in minutes including between a.m. and p.m.
- Solve for unknowns in all places (start time, end time, time interval/elapsed time)

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.NF.2 Understand and represent fractions on a number line **3.MD.4** Measure lengths with halves and fourths of an inch **3.NBT.2** Fluently add and subtract	**4.MD.1** Know relative sizes of hours, minutes, and seconds. Express hours as minutes or seconds and minutes as seconds **4.MD.2** Solve word problems involving intervals of time

Critical Background Knowledge

- Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (2.MD.7)
- Skip-count by fives (2.NBT.2)
- Represent whole numbers on a number line (2.MD.6)
- Understand and tell time on analog and digital clocks to the hour and half hour (1.MD.3)

Academic Vocabulary

minute hand, hour hand, nearest minute, a.m., p.m., midnight, noon, elapsed time, time interval, number line

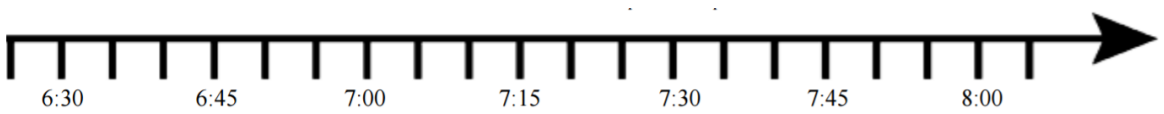
Suggested Models	Suggested Strategies
Example: At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.  	● Apply time to real world situations (class schedule, school events, etc.) ● Connect number lines to the analog clock by viewing a circular clock unfolded into a straight number line ● Connect start time, end time, and time interval (elapsed time) to the number line ● Determine the intervals and sizes of jumps on a number line (hour, half hour, quarter hour, five minute, one minute)

Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/3.pdf

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (Standards 1–2).

Standard 3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). (Excludes compound units such as cubic centimeters [cc or cm3] and finding the geometric volume of a container.) *Add, subtract, multiply, or divide to solve one-step word problems involving masses of objects or volumes of liquids that are given in the same units, for example, by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems.)*

Concepts and Skills to Master

- Understand mass and weight as how heavy or light an object is
- Measure mass/weight of objects in standard units using spring scales, balance scales, and digital scales
- Understand liquid volume and capacity as how much space an amount of liquid takes up
- Measure volume of liquids in standard units using measuring cups, beakers, etc.
- Know relative sizes using benchmarks and mental images of grams (g), kilograms (kg), and liters (l)
- Solve one-step word problems involving measurement units with mass and liquid volume
- Understand conservation of matter and how it impacts estimation of liquid volume (different shaped vessels with the same capacity)

Teacher Note: The core standards do not differentiate between weight and mass. Scientifically for example, mass is the amount of matter in an object while weight is the force exerted on the body of gravity. On the earth's surface, the distinction is not important. Therefore, mass and weight may be used interchangeably in solving measurement problems related to the standard. Students may be, but are not expected to be exposed to the following units not explicitly listed in the core standards: fluid ounces, cups, pints, quarts, gallons, pounds, ounces.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.MD.1 Solve word problems involving addition and subtraction of time intervals in minutes **3.OA.8** Solve two-step word problems using the four operations	**4.MD.1** Know relative sizes of measurement units and express units in a larger unit in terms of a smaller unit using a two-column table **4.MD.2** Solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money **5.MD.1** Use unit conversions in solving multi-step, real world problems

Critical Background Knowledge

- Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes (2.MD.1)
- Estimate lengths using units of inches, feet, centimeters, and meters (2.MD.3)
- Describe measureable attributes of objects and directly compare measureable attributes of two objects (K.MD.1–2)

Academic Vocabulary

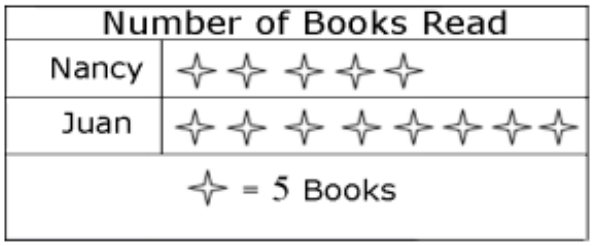
Liquid volumes: liquid volume, capacity, liter (l), measuring cup, beaker, estimate
Masses of objects: mass, weight, kilogram (kg), gram (g), spring scale, balance scale, digital scale, estimate

Suggested Models	Suggested Strategies
	● Compare weights of items by holding an item weighing 1 kg and an item weighing 1 g ● Brainstorm events where exact measurement is necessary and times when an estimate is sufficient ● Identify common items labeled with mass and liquid volume (drink containers, food packages, etc.) ● Develop benchmark references by weighing objects of exactly 1 kg (a 1 kg bag of rice) and 1 g (a centimeter cube) ● Develop benchmark references by measuring liquids of volumes exactly 1 liter (juice bottle)

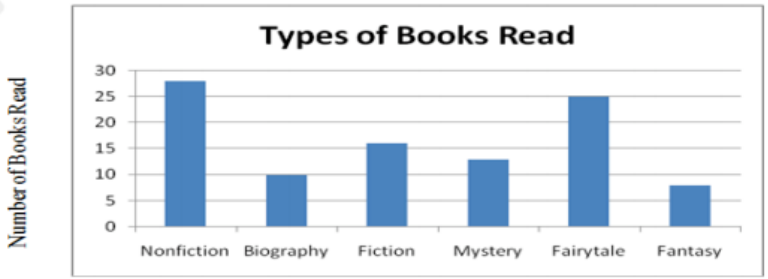
Image Source: https://www.illustrativemathematics.org/content-standards/3/MD/A/2/tasks/1929

Represent and interpret data (Standards 3.MD.3–4).	
Standard 3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent five pets.*	
Concepts and Skills to Master	
• Draw a scaled picture and scaled bar graph to represent data, with several categories • Solve one and two-step problems using data from the scaled bar graph Teacher Note: The Standards in Grades 1–3 do not require students to gather categorical data, just to represent it. Gathering data may be used as an instructional strategy, but it is not required of students. Third Grade is the first time students make scaled graphs.	
Related Standards: Current Grade Level	Related Standards: Future Grade Level
3.0A.3 Solve and Represent Two-Step Word Problems	Standards in future grade levels are more focused on numerical data rather than categorical data
Critical Background Knowledge	
• Draw picture and bar graph (2.MD.10) • Organize, represent and interpret data (1.MD.4)	
Academic Vocabulary	
data, picture graph, bar graph, symbol, key, scaled, category, title labels, compare, how many more/less	
Suggested Strategies	
• Present clear data sets for students to draw a scaled bar graph • Collect or give information to create horizontal and vertical bar graphs and picture graphs • Ensure each student has the opportunity to explain analyze and interpret data	
Suggested Models	

Pictograph: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



Single Bar Graph: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Analyze and Interpret data:

- How many more nonfiction books were read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?

Image Source: http: //www.dpi.state.nc.us/docs/curriculum/mathematics/scos/3.pdf

Represent and interpret data (Standards 3.MD.3–4).

Standard 3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters.

Concepts and Skills to Master

- Measure lengths of several objects to the half inch and quarter inch
- Make a line plot using generated measurements; include a horizontal scale, title, labels, and straight columns of symbols (• or **X**) to represent the data points Make a line plot using generated measurements; include a horizontal scale, title, labels, and straight columns of data marks (For example: dot or X)
- Understand line plots represent measurement data, not categorical data
- Relate line plots to number lines, including representing fractions on a number line
- Teacher Note: Students do not have to generate the data each time they make line plots. That would be too time consuming. After some experiences in generating data, most work in producing line plots can be done by providing students with data sets. While scaffolds may be in place to support students in creating line plots when appropriate, students are expected to create the horizontal scale with tick marks when making line plots. While the emphasis of this standard is on generating data and making line plots, students can pose and answer simple questions about the data, such as how many students obtained measurements larger than 14 ½ inches.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.NF.1 Understand that a unit fraction has a numerator of one and a non-zero denominator. **3.NF.2** Understand and represent fractions on the number line	**4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, and eighths). Solve problems involving addition and subtraction with like denominators of fractions by using information presented in line plots **5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, eighths). Use operations on fractions for this grade to solve problems involving information presented in line plots

Critical Background Knowledge

- Measure the length of an object using whole units (2.MD.1)
- Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2… Represent whole number sums and differences within 100 on a number line diagram. (2.MD.6)
- Generate measurement data and make line plots using whole number units (2.MD.9)

Academic Vocabulary

line plot, data, length, whole, half, quarter, fourth, inch (in.), ", ½", ¼", 2/4", ¾", tick mark, measurement scale

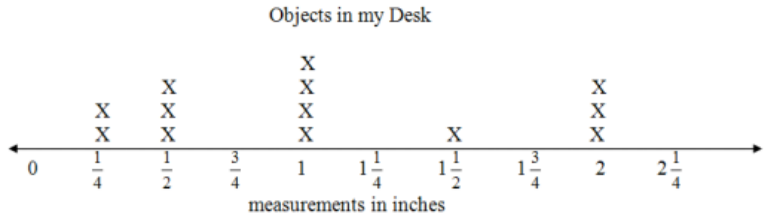
Suggested Models	Suggested Strategies
Example: Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼? ½? etc…  	• Use data tables to record measurements prior to creating a line plot • Generate ideas about what measurement data could be generated and represented on a line plot • Measure physical objects or distances varying in length; use data to create a line plot

Image Source: https://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf

Understand concepts of area and relate area to multiplication and addition (Standards 3.MD.5–7).

Standard 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length one unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Concepts and Skills to Master

- Students recognize area as an attribute of two-dimensional regions
- Understand “a unit square” and “one square unit” in relation to area
- Measure the area by finding the total number of same size units to cover the shape without gaps or overlaps

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.MD.6 Measure area by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units) **3.MD.7** Relate area to the operations of multiplication and addition **3.MD.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters **3.OA.5** Apply properties of operations to multiply and divide	**4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems **4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers. Illustrate and explain the calculation by using area models **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Illustrate and explain the calculation by using area models **5.NF.4. b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths **5.MD.3** Recognize volume as an attribute of solid figures and understand the concepts of volume measurement

Critical Background Knowledge

- Compose two-dimensional shapes to create composite shapes (1.G.2)
- Measure the length of an object by selecting and using appropriate tools (2.MD.1)

Academic Vocabulary

area, attribute, plane figure, unit square, a square unit, gaps, overlaps, side length

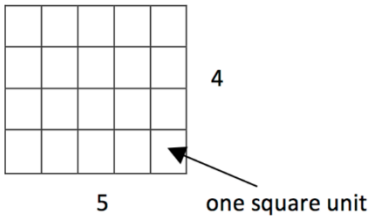
Suggested Models:	**Suggested Strategies:**
	• Explore the concept of covering or tiling a region with “unit squares” which could include square tiles or shading on grid or graph paper. • Students should have ample experiences filling a region with square tiles before transitioning to pictorial representations on grid paper.

Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/3.pdf

3.MD.5

Understand concepts of area and relate area to multiplication and addition (Standards 3.MD.5–7).

Standard 3.MD.6 Measure area by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units).

Concepts and Skills to Master

- Identify square units
- Count the square units to find the area

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement **3.MD.7** Relate area to the operations of multiplication and addition **3.OA.5** Apply properties of operations to multiply and divide	**4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems **5.NF.4. b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths **5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world mathematical problems

Critical Background Knowledge

- Partition rectangles into rows and columns of same-size squares and count to find the total number of them (2.G.2)
- Understand the relationship between numbers and quantities; connect counting to cardinality (K.CC.4)

Academic Vocabulary

area, array, square unit, square, square centimeter, square inch, square meter, square feet, ft2, ft, m, in

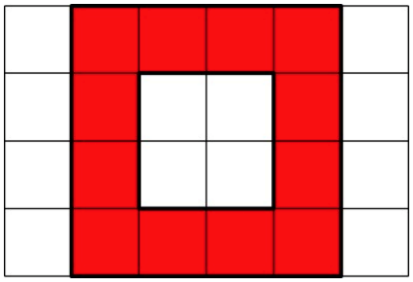
Suggested Models	Suggested Strategies
Find the area of the colored figure. 	• Count the square units to find the area (This should be done in metric, customary, and non-standard square units) • Use different sized grid paper or 12x12 paper to explore the areas measured in square centimeters, square inches and square feet

Image Source: http://s3.amazonaws.com/illustrativemathematics/attachments/000/009/306/original/public_task_1515.pdf?1462396036

Understand concepts of area and relate area to multiplication and addition (Standards 3.MD.5–7).

Standard 3.MD.7 Relate area to the operations of multiplication and addition (refer to 3.OA.5).

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Concepts and Skills to Master

- Recognize area as additive
- Use tiling to find the area of a rectangle using whole numbers
- Understand and explain why multiplying side lengths of a rectangle is the same as counting the tiles
- Use real-world problems/context that multiply side lengths to find area using whole numbers
- Use the area model to represent the distributive property
- Understand and explain that the area of a rectangular region can be found either by multiplying the side lengths (5 x 8) or by adding two products (5 x 2)+ (5 x 6) which illustrates the distributive property
- Decompose rectilinear figures into rectangles, find the area of each part then add the areas of the various rectangles together

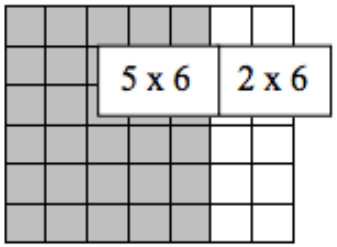
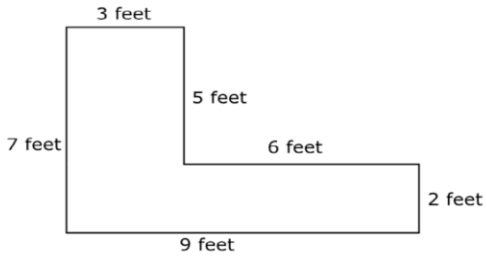
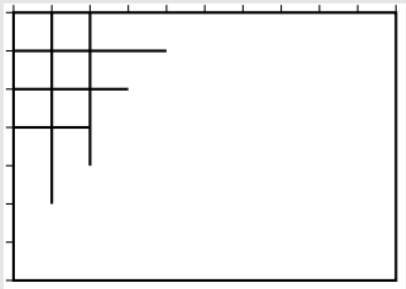
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement **3.MD.6** Measure area by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units). **3.MD.8** Solve real-world and mathematical problems involving perimeters of polygons, exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters	**4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems **5.NF.4. b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes;

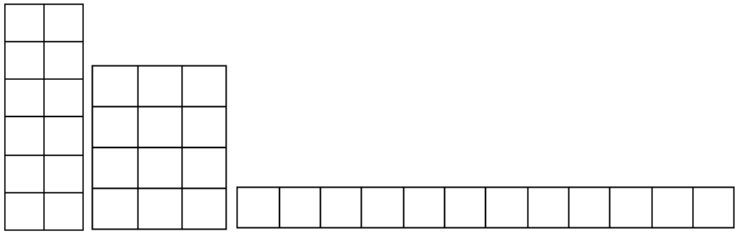
Critical Background Knowledge

- See Related Standards: Current Grade Level
- Partition rectangles into rows and columns of same-size squares and count to find the total number of them (2.G.2)

Academic Vocabulary

area, tiling, product, additive, distributive property, rectilinear, decompose

Suggested Models	Suggested Strategies
Tile the rectangle and then multiply the side lengths to show it is the same. To find the area one could count the squares or multiply 3 x 4 = 12. 1 2 3 4 5 6 7 8 9 10 11 12 This standard extends students' work with the distributive property. For example, in the model below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.  Find the area of the figure in square feet by decomposing the figure into rectangles and adding the areas of the rectangles.  	• Use square tile to tile a rectilinear figure; count, skip count, or multiply and/or add to find the total number of tiles • Relate skip counting to multiplication to calculate the area of a rectilinear figure **Incomplete array**  To determine the area of this rectangular region, students might be encouraged to construct a row, corresponding to the indicated positions, then repeating that row to fill the region. Cutouts of strips of rows can help the needed spatial structuring and reduce the time needed to show a rectangle as rows or columns of squares. Drawing all of the squares can also be helpful, but it is slow for larger rectangles. Drawing the unit lengths on the opposite sides can help students see that joining opposite unit end-points will create the needed unit square grid.
Image Sources:http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/3.pdf, https://commoncoretools.files.wordpress.com/2012/07/ccss_progression_gm_k5_2012_07_21.pdf	

Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures (Standard 3.MD.8).	
Standard 3.MD.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	
Concepts and Skills to Master	
• Solve real-world and mathematical problems involving perimeter • Find the perimeter given the side lengths • Find an unknown side length given the perimeter • Find rectangles with the same perimeter and different area • Find rectangles with the same area and different perimeters	
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement **3.MD.6** Measure area by counting unit squares **3.MD.7** Relate area to the operations of multiplication and addition **3.OA.8** Solve two-step word problems using the four operations using whole numbers	**4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems **5.NF.4. b.** Find the area of a rectangle with fractional side lengths
Critical Background Knowledge	
• Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions (2.OA.1) • Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4) • See Related Standards: Current Grade Level	
Academic Vocabulary	
polygon, side length, area, perimeter, linear, plane figure	
Suggested Models	Suggested Strategies
 Each rectangle has an area of 12 square units, but the perimeters are 16 units, 14 units, and 26 units.	• Walk around the perimeter of a room discussing the measurements • Use rubber bands to represent the perimeter of a polygon on a geoboard or trace around a polygon on a whiteboard • Use addition to find perimeters; recognize the patterns that exist when finding the sum of the lengths and widths of rectangles • Use graph paper or square tiles to create rectangles with the same perimeter and different areas or with the same area and different perimeters, justify claims • Find the perimeters of all rectangles with an area of 12 square units
Image Source : http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/3.pdf	

Reason with shapes and their attributes (Standards 3.G.1–2).

Standard 3.G.1 Understand that shapes in different categories *(for example, rhombuses, rectangles, and others)* may share attributes *(for example, having four sides)*, and that the shared attributes can define a larger category *(for example, quadrilaterals).* Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Concepts and Skills to Master

- Understand that squares, rectangles, rhombuses, parallelograms, and trapezoids are examples of quadrilaterals
- Compare and contrast squares, rectangles, rhombuses, parallelograms, and trapezoids
- Identify and draw quadrilaterals that cannot be classified as squares, rectangles, rhombuses, parallelograms, or trapezoids
- Recognize and understand that the larger category of quadrilaterals includes other subcategories such as squares, rectangles, rhombuses, parallelograms, and trapezoids; Identify examples and non-examples of squares, rectangles, rhombuses, parallelograms, and trapezoids; Recognize that there are quadrilaterals that are not in any of the subcategories

Teacher Note: While students are expected to informally recognize attributes of quadrilaterals, including parallel lines and right angles, they are not expected to master these concepts until fourth grade. Developing a hierarchy of quadrilateral shapes is reserved for fifth grade (5.G.4). In third grade, students only make basic connections between the attributes of these shapes. Note that in the U.S., that the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. Both definitions are accepted in the United States. Utah has adopted the inclusive definition. **A trapezoid is a quadrilateral with at least one pair of parallel sides.** The inclusive definition is the most accepted definition worldwide and is the definition used by the Utah State Board of Education for standard and assessment purposes.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.G.2 Partition shapes into parts with equal areas	**4.G.1** Draw points, lines, line segments, rays, angles (right, acute, and obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures **4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.4.G.1 **4.MD.5** Recognize angles as geometric figures **5.G.3** Understand that attributes belonging to a category of two-dimensional figures all belong to all subcategories of that category **5.G.4** Classify two-dimensional figures in a hierarchy based on properties

Critical Background Knowledge from Previous Grade Levels

- Recognize and draw shapes having specified attributes, such as a given number of sides of angles. Identify and describe quadrilaterals, squares, rectangles, and trapezoids (2.G.1)
- Identify and distinguish between defining attributes versus non-defining attributes; build and draw shapes that possess defining attributes (1.G.1)
- Students work with trapezoids, squares, and rectangles in first and second grade. The term *quadrilateral* is introduced in second grade
- Students work with squares, circles, triangles, rectangles, and hexagons in Kindergarten

Academic Vocabulary
attribute, angle, closed figure, open figure, parallel ||, side, polygon, quadrilateral, rhombus, rectangle, square, parallelogram, trapezoid right angle ⌐, corners Shapes new to third grade: rhombus, parallelogram Teacher Note: Rectilinear figures must have four right angles. Ensure that correct plural forms of vocabulary words are used. The plural form for rhombus may be rhombuses or rhombi (may be used interchangeably). The plural form for vertex is vertices.

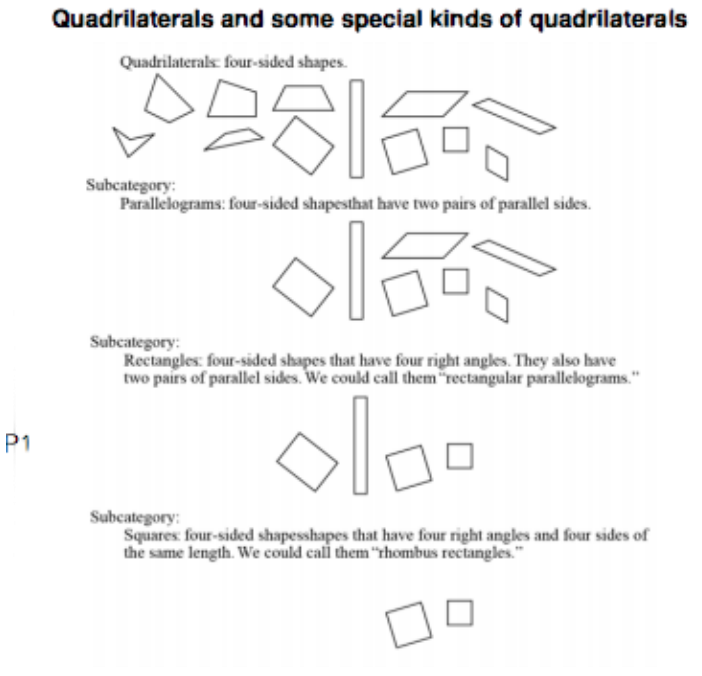
Suggested Models	Suggested Strategies
 The representations above might be used by teachers in class. Note that the left-most four shapes in the first section at the top left have four sides but do not have properties that would place them in any of the other categories shown (parallelograms, rectangles, squares).	• Analyze collections of each shape (quadrilateral, trapezoid, parallelogram, rectangle, rhombus, and square) to determine the defining attributes; compare and contrast the attributes of several different shapes • Create or represent many varied and unusual squares, rectangles, rhombuses, parallelograms, and trapezoids and explain them verbally or in written form; students also create or represent examples of quadrilaterals that do not belong in any of the subcategories • Use graphic organizers to categorize sets of shapes • Draw shapes as examples and non-examples in given categories and subcategories

Image Source: http://commoncoretools.me/wp-content/uploads/2014/12/ccss_progression_gk6_2014_12_27.pdf pp.13

Reason with shapes and their attributes (Standards 3.G.1–2).

Standard 3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into four parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Concepts and Skills to Master

- Partition shapes into parts with equal areas ($\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$)
- Understand the denominator of the fraction as the fractional name determined by the number of pieces in the whole (for example, when a whole shape is partitioned into six equal parts, the fractional parts are sixths)
- Understand that the parts must have equal areas in order to use fractional notation to describe their size
- Understand that one of the equal parts is a unit fraction (when a shape is partitioned into 6 equal parts, one of the parts is $\frac{1}{6}$)

Teacher Note: Third grade is the first time students work with fractions as numbers, including fractional notation as well as representations beyond circles and rectangles. Equal shares, equal areas, and equal parts may be used interchangeably.

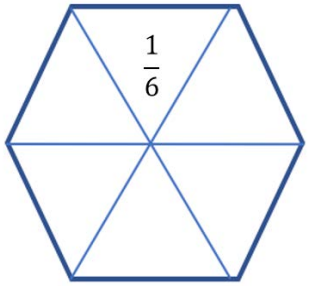
Related Standards: Current Grade Level	Related Standards: Future Grade Level
3.NF.1 Understand unit fractions **3.NF.2** Understand a fraction as a number on the number line **3.NF.3** Explain equivalence of fractions and compare fractions by reasoning about their size **3.MD.6** Measure area by counting unit squares	**4.NF.3** Understand fractions as sums of unit fractions **4.NF.1, 4.NF.2, 4.NF.4, 5.NF.1, 5.NF.2, 5.NF.4, 5.NF.6, 5.NF.7** Use area models to represent operations with fractions **4.G.3** Recognize and draw a line of symmetry

Critical Background Knowledge from Previous Grade Levels

- Partition rectangles into rows and columns and count to find the total (2.G.2)
- Partition circles and rectangles into two, three, and four equal shares; describe the shares as halves, thirds, fourths, and quarters. Recognize that equal shares of identical wholes need not have the same shape (2.G.3, 1.G.3)
- Notice smaller shapes within a larger existing shape (1.G.2)

Academic Vocabulary

partition, fraction, unit fraction, whole, area, equal area, numerator, denominator, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$

Suggested Models	Suggested Strategies
	• Practice paper folding and identify each equal part with the fractional notation • Partition regions into equal shares using a context and name the shares using fractional notation (cookies, pies, pizza, brownies, crackers, grass area, etc.) • Sort shapes that are partitioned into equal shares and shares that are not equal • Partition shapes using manipulatives such as geoboards, pattern blocks, and paper rectangles and circles, food, etc.