Guidance for Acceleration in Utah's Mathematics Pathways

The purpose of this document is to provide guidance for all stakeholders around the idea of mathematics pathway acceleration. Acceleration deserves thoughtful attention to ensure that students are challenged and that they are mastering the full range of mathematical content and skills, without omitting critical standards. Acceleration is defined as moving at a faster rate through a defined mathematics pathway than grade level peers. Mathematics pathways are a set of courses designed to prepare students for post-secondary success.

Two resources that are instrumental in considering the effectiveness of an accelerated pathway are Utah's Core State Standards for Mathematics and The Standards for Mathematical Practice, which span all grade levels and are built throughout a student's mathematical journey. The Standards for Mathematical Practice are the skills that mathematicians rely upon to reason and make mathematical sense and should be intentionally explicit in all K-12 mathematics experiences including accelerated mathematics courses and pathways:

- Standard 1: Make sense of problems and persevere in solving them
- Standard 2: Reason abstractly and quantitatively
- Standard 3: Construct viable arguments and critique the reasoning of others
- Standard 4: Model with mathematics
- Standard 5: Use appropriate tools strategically
- Standard 6: Attend to precision
- Standard 7: Look for and make sure of structure
- Standard 8: Look for and express regularity in repeated reasoning

Both the Utah Core State Standards for Mathematics and The Standards for Mathematical Practice intended to empower students to meet the aspirations of Utah's Portrait of a Graduate. The standards define what students should know and be able to do in mathematics at each grade level through building adaptive reasoning, strategic competence, conceptual understandings, productive dispositions, and procedural fluency. The math standards provide both the challenge and the depth necessary for all students to engage meaningfully in mathematics.

What does identification of students who would benefit from an accelerated mathematics pathway look like?

Care should be taken to properly recognize students who could benefit from an accelerated pathway as outlined in R277-707-3. This should be done through multiple assessment measures and with the understanding that the number of students who would benefit from an accelerated mathematics pathway typically does not exceed 5-7% of a schools' overall population. The 5-7% who would benefit from an accelerated mathematics pathway should mirror the demographic representation of the school (including but not limited to ethnicity, socio-economic status, multilingual learners, and students who receive Special Education services). LEA's have autonomy to determine a system that best meets the needs of their local population, including making local decisions about running stand-alone accelerated mathematics classes or offering enrichment within the foundational mathematics course.

All LEAs should have a process for identifying and serving students whose academic achievement would benefit from the support of acceleration. This process should include multiple measures and should accommodate for disabilities, potential language barriers, culturally diverse perspectives, and should not be dependent on a student's English vocabulary or comprehension skills. A universal screening practice should be

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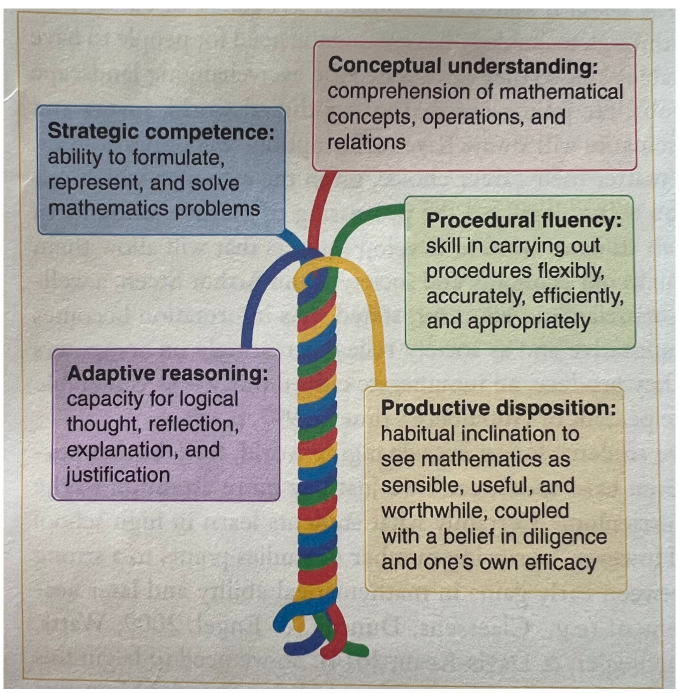
in place to ensure that every student has equal access to qualification for acceleration. At a minimum, this should include an LEA analyzing end of level summative assessment data (RISE or Aspire) for instances of students scoring “highly proficient” with the understanding that summative assessment data can be used as a starting point for identification and further data points must be gathered In accordance with [R277-707](). Additionally, students and their families should engage in conversations with counselors regarding potential qualification for accelerated coursework and about implications on the student’s [mathematics pathways]() and how that pathway aligns with and supports a student’s future career aspirations--keeping in mind that acceleration in mathematics should open up opportunities for students to engage in mathematics courses in High School that align with a student’s future aspirations and should not be used so that students have a “gap year” of mathematics at any point in their secondary mathematics pathway.

What is an accelerated student/gifted student/high ability student in mathematics?
An accelerated student in mathematics has strong adaptive reasoning, strategic competence, conceptual understandings, productive dispositions, and procedural fluency related to grade-level standards. When students have [demonstrated mastery]() of grade-level core standards (which includes the Standards for Mathematical Practice), they may choose to participate in an accelerated mathematics course. Accelerated courses must not skip any content or reduce rigor. Instead, they should move at a faster pace and include multiple assessments to ensure that the [content and the Standards for Mathematical Practice have been mastered]().

Identifying students who display talent with Mathematical proficiency

Mathematical proficiency is not just fact fluency and recall, it includes five interwoven components as illustrated below:



(Kilpatrick, et. al, 2001)

Students who would benefit from an accelerated mathematics pathway are students who demonstrate competency in all five components of mathematical proficiency: adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency.

Adaptive Reasoning

Mathematics is more than a set of rules and procedures. Mathematical proficiency includes the ability to justify and think logically about problems. Students who demonstrate the capacity to solve problems flexibly and think through their solutions and outcomes have the *adaptive reasoning* necessary to be proficient in mathematics. "In mathematics, adaptive reasoning is the glue that holds everything together, the lodestar that guides learning." (Kilpatrick, et. al, 2001)

Strategic Competence

Students who are able to formulate, make sense of, and solve mathematical problems demonstrate *strategic competence* in mathematics. Students who demonstrate strategic competence are adept at thinking logically to derive multiple solution pathways for a variety of cognitively deep mathematical problems. Students understand there is not only one method only for solving mathematical problems and work to find and understand the multiple methods and models for arriving at a solution.

Conceptual Understanding

Conceptual understanding is necessary in order for students to develop a mathematical foundation and is essential for developing procedural fluency. Conceptual understanding is defined as the "comprehension and connection of concepts, operations, and relations" (NCTM, 2014, p. 7) i.e. the ability to apply mathematical procedures in multiple contexts.

Productive Disposition

A student's attitude towards mathematics is a major contributor to their educational success or failure. Students with a *productive disposition* around mathematics see math as sensible, useful, and most importantly, see themselves as learners and doers of mathematics through the Standards for Mathematical Practice:

- Standard 1: Make sense of problems and persevere in solving them
- Standard 2: Reason abstractly and quantitatively
- Standard 3: Construct viable arguments and critique the reasoning of others
- Standard 4: Model with mathematics
- Standard 5: Use appropriate tools strategically
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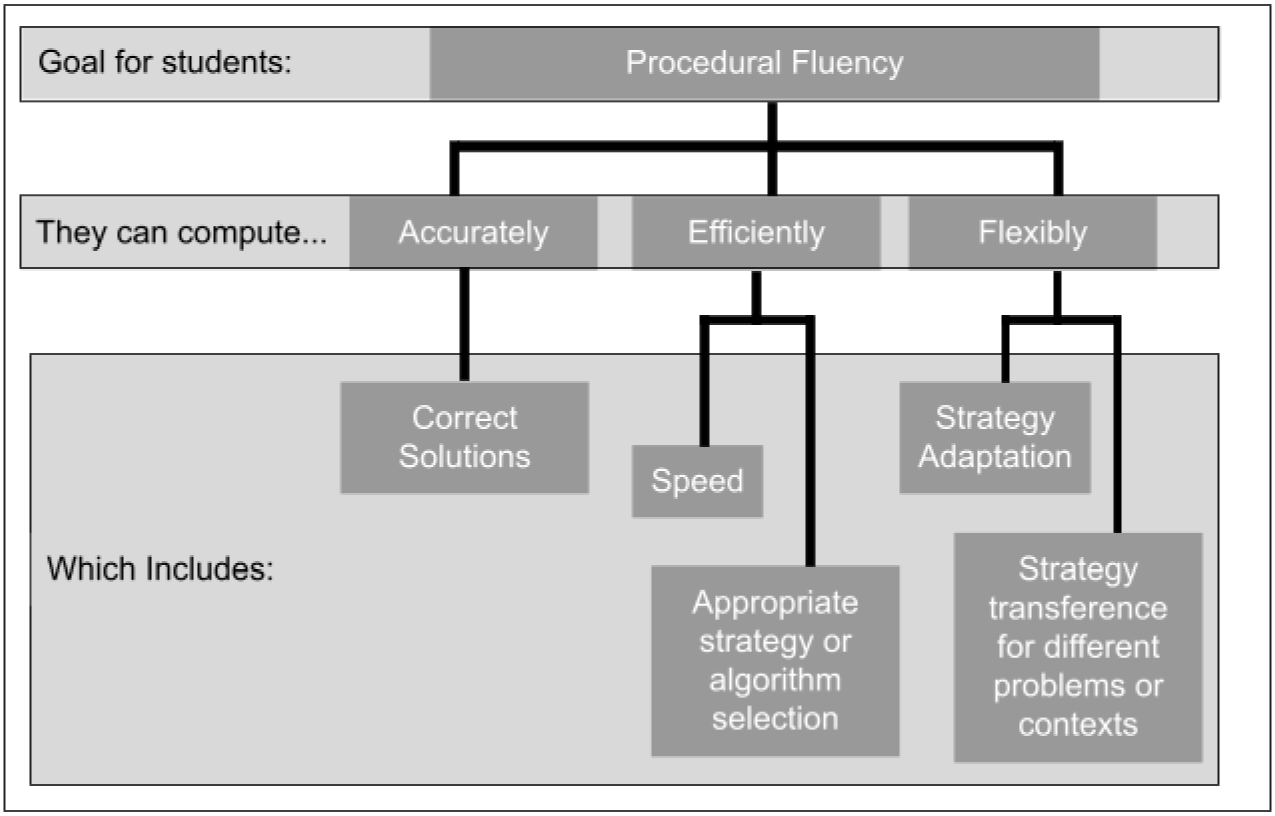
A productive disposition means a willingness to engage in productive struggle with mathematical problems and to seek out and learn from challenging situations. (Kilpatrick, et. al, 2001)

Procedural Fluency

Procedural fluency refers to knowledge of procedures, knowledge of when and how to use procedures appropriately, and skill in performing procedures flexibly, accurately, and efficiently (Kilpatrick, et. al., 2001). Developing procedural fluency goes beyond memorization of facts or a list of procedures that are not connected to an understanding of "why it works". (Baroody 2006; Griffin 2005). Additionally, "procedural fluency...is fragile and meaningless without a sound conceptual understanding of the mathematics" (NCTM, 2017, p. 55) as summarized in the diagram below:



(Spangler & Wanko, 2017, p. 63)

Conceptual understanding and procedural fluency work together to help students develop strategic competence (i.e., the ability to formulate, represent, and solve mathematical problems) and adaptive reasoning (i.e., the capacity to think logically and to justify one's thinking). These competencies are both necessary for students when solving mathematics problems that they may encounter in real life, as well as within mathematics and other disciplines. (NCTM, 2014, p. 7). Students who would benefit from an accelerated mathematics pathway are students who demonstrate competency in all five components of mathematical proficiency: adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency.

Organization of an Accelerated Mathematics Program

In circumstances where acceleration is appropriate, there must be coherence in the system so that students have pathways to opportunity as opposed to pathways to nowhere. LEA's and schools should use course codes that match the Utah Core Standards that the students are studying, even if that means using multiple course codes within one school year (which also may include more than one summative assessment in a given school year). Educators who teach accelerated mathematics content should hold all proper licensing and endorsements for the level of mathematics they are teaching.

An accelerated mathematics pathway should happen in conjunction with a systemic plan and should occur after sixth grade (at the earliest) with an understanding that productive student outcomes are attained when acceleration happens later in a student's mathematical pathway rather than sooner (Boaler, 2016; Illustrative Mathematics, 2020).

In the accelerated mathematics pathway, "..educators must ensure that accelerated opportunities are appropriate and no critical concepts are rushed or skipped" (Leinwand & Milou, 2021, p. 80). Differentiated instruction and customized supports for accelerated students should be considered thoughtfully and implemented in line with the Utah Multi-Tiered System of Supports for Mathematics (UMTSS). Tier 2, as referenced in UMTSS, should be seen as a high-impact way to extend and deepen mathematical learning opportunities for accelerated students. Task-based instruction, which naturally differentiates content for students, should be implemented intentionally and consistently in an accelerated mathematics classroom and with accelerated mathematics students (NCTM, 2014). Teachers of accelerated mathematics students should take care to implement the high-leverage Mathematics Teaching Practices:

1. Establish mathematics goals to focus learning,
2. Implement tasks that promote reasoning and problem solving,
3. Use and connect mathematical representations,
4. Facilitate meaningful mathematical discourse,
5. Pose purposeful questions,
6. Build procedural fluency from conceptual understanding,
7. Support productive struggle in learning mathematics, and
8. Elicit and use evidence of student thinking.
   (NCTM, 2014, p. 10)

As Board Rules 277-707 suggests, students' varying needs can be met through "opportunities with increased depth, complexity, or rigor" in addition to acceleration. The vast majority of student needs can be met through providing differentiated instruction in a general education mathematics classroom with low threshold, high ceiling tasks aligned with learning goals. Ensuring these classrooms are effectively serving students with a wide array of mathematical backgrounds and achievement levels should be pursued in concert with acceleration strategies to ensure each system has in place a coherent and universal math pathways system for its students. Teachers, specialists and administrators should all have a voice as LEAs make decisions about how best to design these local pathways and should offer clear communication to families in order to meet the needs of the students they serve.

References

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