

PRECALCULUS

Prerequisite: Secondary III

THE MAIN GOAL OF PRECALCULUS IS FOR STUDENTS TO GAIN A DEEP UNDERSTANDING OF THE FUNDAMENTAL CONCEPTS AND RELATIONSHIPS OF FUNCTIONS. Students will expand their knowledge of quadratic, exponential, and logarithmic functions to include power, polynomial, rational, piece-wise, and trigonometric functions. Students will investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use graphing calculators and mathematical software to build understanding, make connections between representations, and provide support in solving problems. Students will analyze various representations of functions, sequences, and series. Students will analyze bivariate data and data distributions. Students will apply mathematical skills and make meaningful connections to life's experiences. Precalculus is highly recommended preparation for students who plan to continue their formal education beyond high school.

Standard I: Students will use the language and operations of algebra to evaluate, analyze and solve problems.

Objective 1: Compute with matrices and use matrices to solve problems.

- Represent real-world situations with matrices.
- Add, subtract, and multiply (including scalar multiplication) matrices using paper and pencil, and computer programs or calculators.
- Demonstrate that matrix multiplication is associative and distributive, but not commutative.
- Determine additive and multiplicative identities and inverses of a matrix when they exist.
- Solve systems of linear equations with up to three variables using matrices.

Objective 2: Analyze the behavior of sequences and series.

- Describe a sequence as a function where the domain is the set of natural numbers.
- Represent sequences and series using various notations.
- Identify arithmetic and geometric sequences and series.

Mathematical Language and Symbols Students Should Use: matrix, scalar, sequence, series, arithmetic sequence, arithmetic series, geometric sequence, geometric series, Σ

Standard II: Students will understand and represent functions and analyze function behavior.

Objective 1: Analyze and solve problems using polynomial functions.

- Raise a binomial to a power using the Binomial Theorem and Pascal's Triangle.
- Determine the number and nature of solutions to polynomial equations with real coefficients over the complex numbers.
- Factor polynomials to solve equations and real-world applications.
- Understand the relationships among the solutions of a polynomial equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial.
- Write an equation with given solutions.

Objective 2: Model and graph functions and transformations of functions.

- Model real-world relationships with functions.
- Graph rational, piece-wise, power, exponential, and logarithmic functions.

- c. Identify the effects of changing the parameter a in $y = af(x)$, $y = f(ax)$, $y = f(x-a)$, and $y = f(x) + a$, given the graph of $y = f(x)$.

Objective 3: Analyze the behavior of functions.

- a. Identify the domain, range, and other attributes of families of functions and their inverses.
- b. Approximate instantaneous rates of change and find average rates of change using graphs and numerical data.
- c. Identify and analyze continuity, end behavior, asymptotes, symmetry (odd and even functions), and limits, and connect these concepts to graphs of functions.
- d. Determine intervals over which a function is increasing or decreasing, and describe the intervals using interval notation.
- e. Relate the graphical representation of discontinuities and end behavior to the concept of limit.

Mathematical Language and Symbols Students Should Use:
Theorem, rate of change, interval, asymptote, hole, extrema, discontinuous, continuous, odd and even function, limit, Σ

Standard III: Students will use algebraic, spatial, and logical reasoning to solve geometry and measurement problems.

Objective 1: Solve problems using trigonometry.

- a. Define the six trigonometric functions using the unit circle.
- b. Prove trigonometric identities using definitions, the Pythagorean Theorem, or other relationships.
- c. Simplify trigonometric expressions and solve trigonometric equations using identities.
- d. Solve problems using the Law of Sines and the Law of Cosines.
- e. Construct the graphs of the trigonometric functions and their inverses, and describe their behavior, including periodicity and amplitude.

Objective 2: Graph curves using polar and parametric equations.

- a. Define and use polar coordinates and relate them to Cartesian coordinates.
- b. Represent complex numbers in rectangular and polar form, and convert between rectangular and polar form.
- c. Translate equations in Cartesian coordinates into polar coordinates and graph them in the polar coordinate plane.
- d. Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.
- e. Define a curve parametrically and draw parametric graphs.

Objective 3: Solve problems involving the geometric properties of conic sections.

- a. Write equations of conic sections in standard form.
- b. Identify the geometric properties of conic sections (i.e., vertex, foci, lines of symmetry, directrix, major and minor axes, and asymptotes).
- c. Solve real-world applications of conic sections.

Mathematical Language and Symbols Students Should Use:
Law of Sines, Law of Cosines, conic section, ellipse, hyperbola, secant (sec), cosecant (csc), cotangent (cot), polar coordinates, parametric

Standard IV: Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

Objective 1: Compute probabilities for discrete distributions and use sampling distributions to calculate approximate probabilities.

- a. Obtain sample spaces and probability distributions for simple discrete random variables.
- b. Compute binomial probabilities using Pascal's Triangle and the Binomial Theorem.
- c. Compute means and variances of discrete random variables.

- d. Compute probabilities using areas under the Normal Curve.
- e. Calculate parameters of sampling distributions for the sample average, sum, and proportion.
- f. Calculate probabilities in real problems using sampling distributions.

Objective 2: Analyze bivariate data using linear regression methods.

- a. Fit regression lines to pairs of numeric variables and calculate the means and standard deviations of the two variables and the correlation coefficient, using technology.
- b. Compute predictions of y -values for given x -values using a regression equation, and recognize the limitations of such predictions.
- c. Compute and use the standard error for regression.

Mathematical Language and Symbols Students Should Use:
regression line, correlation coefficient, standard error